GO2: An ALE-enabled ocean model

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GO2: A New Ocean Modelling Framework

NASA-GISS currently employs two ocean models in its coupled climate framework: the HYCOM isopycnal model and the GISS-ER level (mass) coordinate model.

The focus of the GO2 project is to create a new 'flexible' ocean modelling environment combining the best features of HYCOM and GISS-ER – **we're using ALE to achieve this flexibility!**

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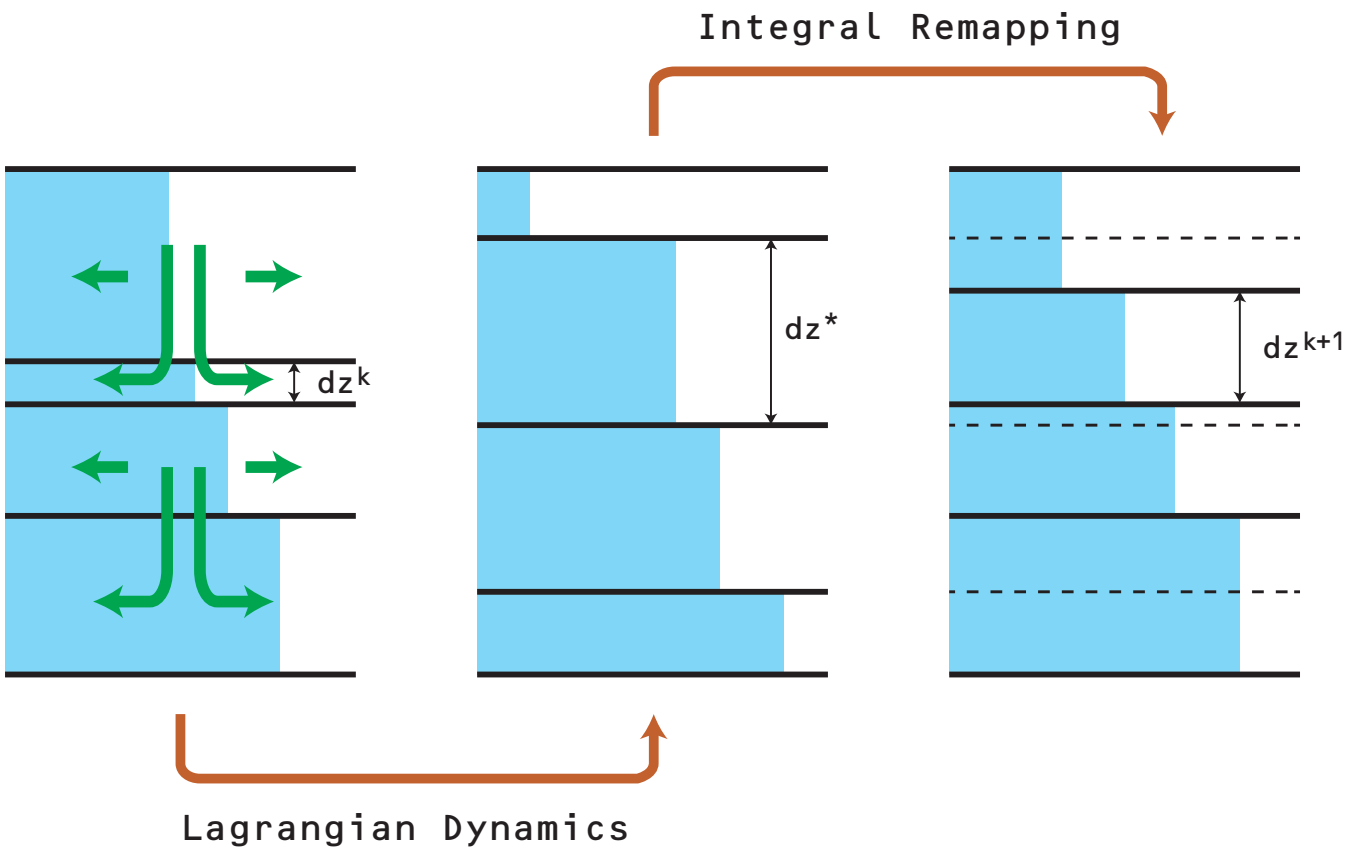
Vertical Dynamics – ALE formulation

We adopt a 'flexible' vertical discretisation, designed to facilitate the use of **arbitrary** vertical coordinates:

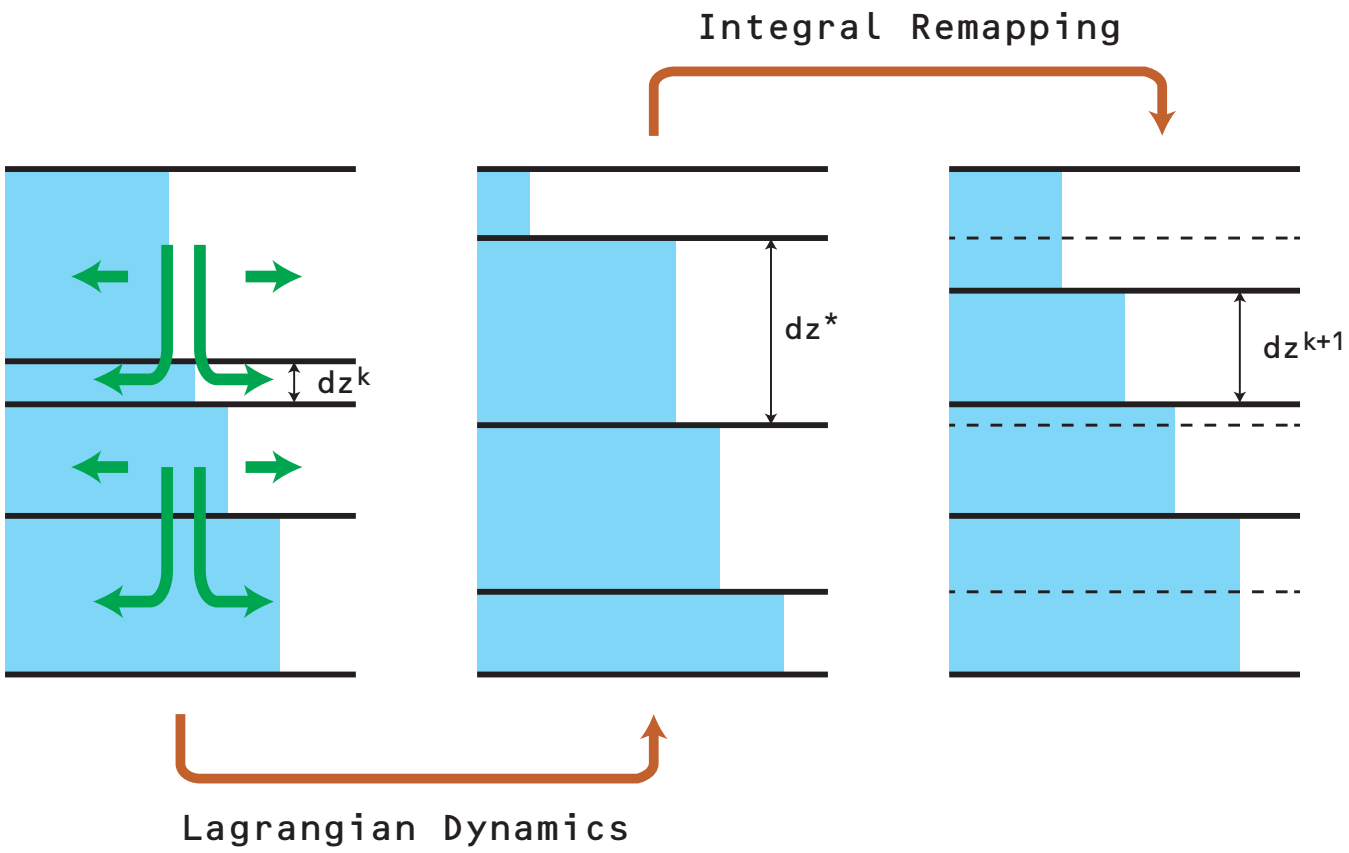
- Supports isopycnal representations (ρ-grid) [HYCOM].
- Supports fixed coordinate systems ('mass'-grid) [GISS-ER].
- Investigate model variability w.r.t 'hybrid' vertical coordinates.



Vertical Dynamics – ALE formulation

ALE-type algorithms separate dynamics into two categories: (i) purely 'Lagrangian' steps for horizontal motion, and (ii) vertical 'remapping' steps (vertical advection) to adjust coordinate surfaces:

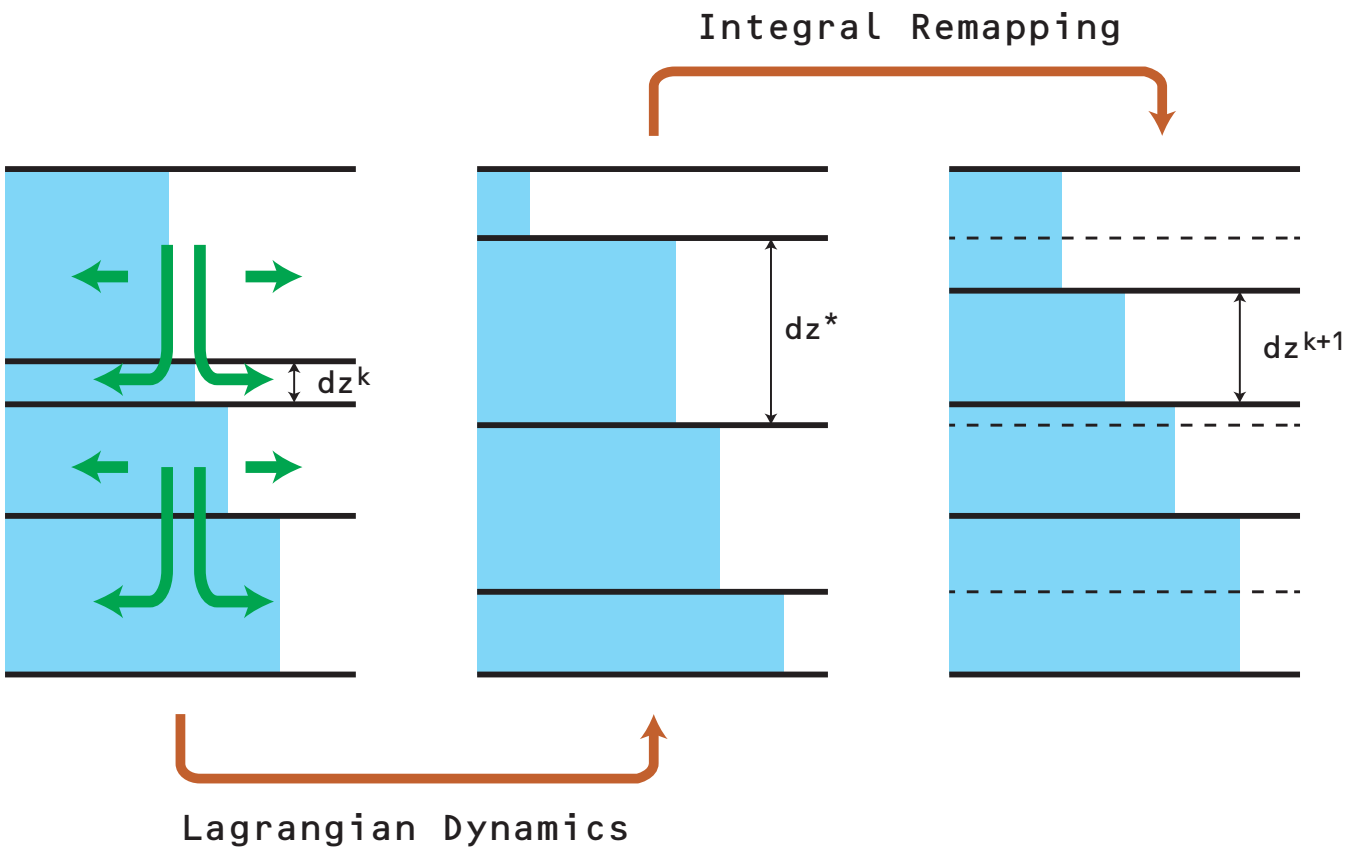
- Lagrangian sub-steps model a 'stack' of shallow-water layers (thickness changes, but only layer-wise transports).



Vertical Dynamics – ALE formulation

Vertical 'remapping' is an inherently flexible way to think about vertical advection – can build different 'models' simply by making various 'choices' w.r.t target coordinate.

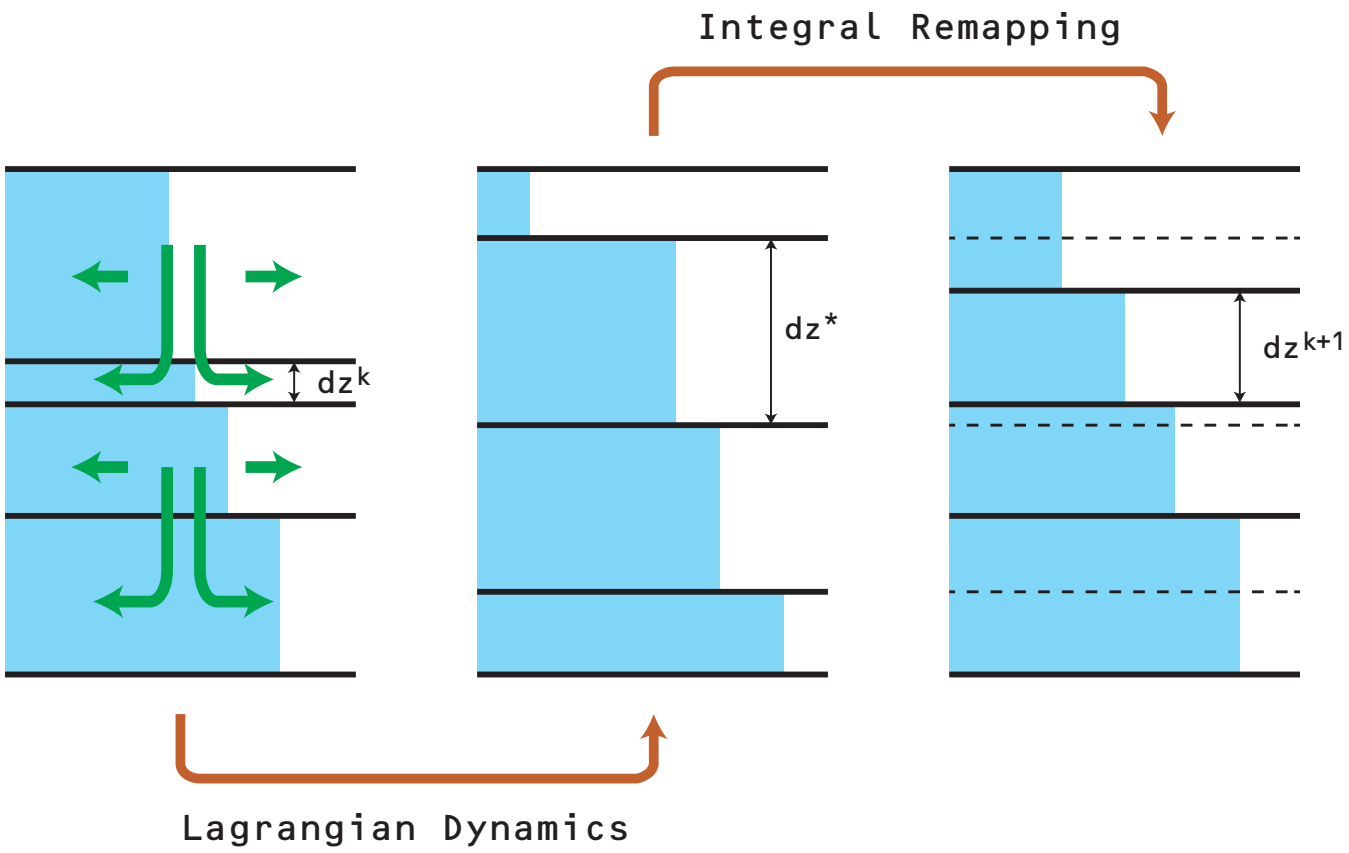
- If remapping is done 'frequently' and the target coordinate is **z** we achieve a model similar to `GISS-ER` / `MITgcm`.



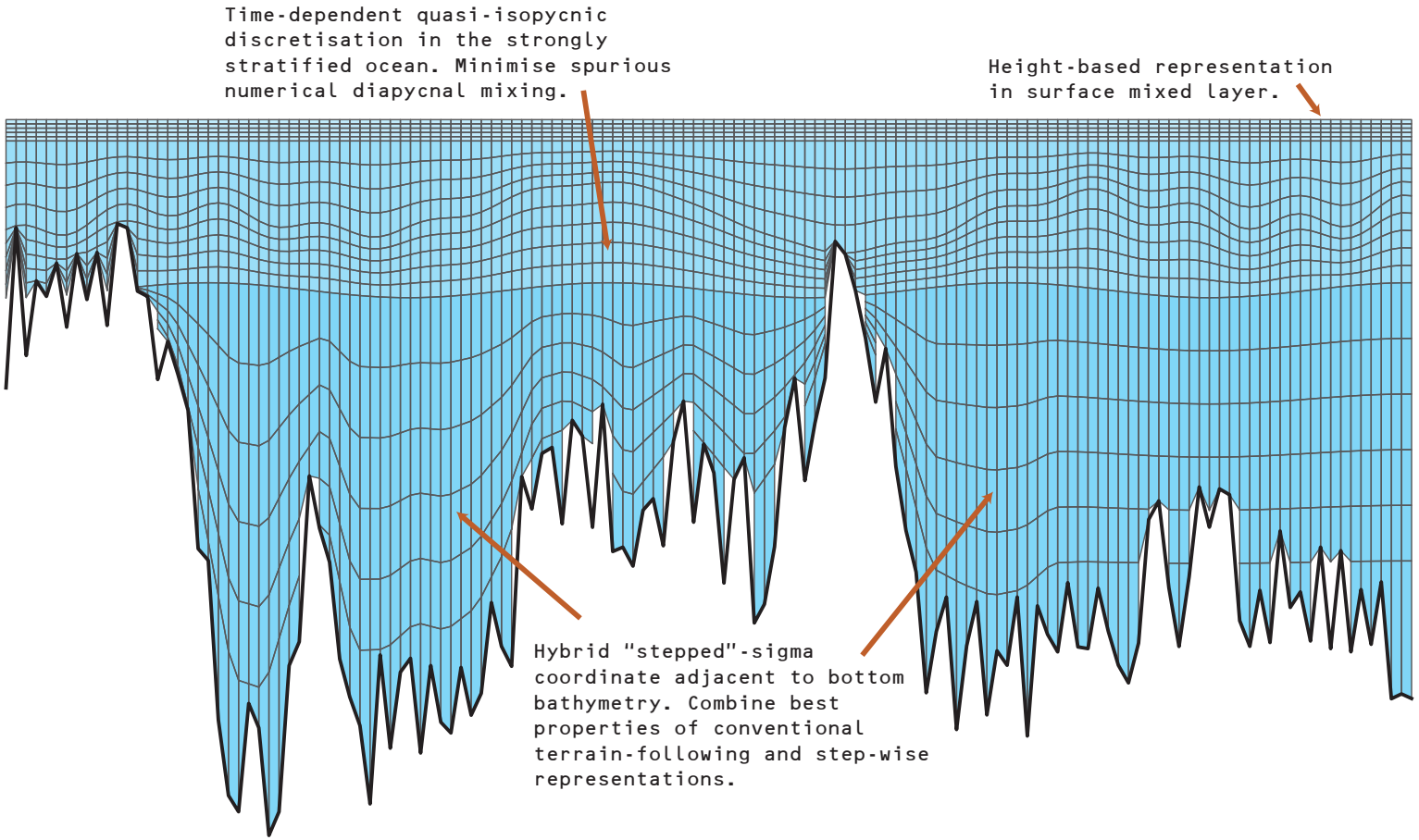
Vertical Dynamics – ALE formulation

Vertical 'remapping' is an inherently flexible way to think about vertical advection – can build different 'models' simply by making various 'choices' w.r.t target coordinate.

- If the target coordinate is quasi-$\rho(S, T, p_0)$ we achieve a model similar to `HYCOM` / `GISS-EH`.



Vertical Dynamics – Target grid selection



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Coupled Dynamics – Full formulation

Expressed formally, we evolve (Lagrangian-ly) a stack of impermeable, non-boussinesq fluid layers on an arbitrary vertical coordinate surface s, with *zero* explicit cross-coordinate fluxes $\dot{s} = 0$:

$$\partial_t(\mathbf{u}_h) + (\mathbf{u}_h \cdot \nabla_s)\mathbf{u}_h + f\mathbf{u}_h^{\perp} = \nabla_s(\Phi) + \rho^{-1}\nabla_s(p) + \mathbf{R}_{\mathbf{u}_h} \tag{1}$$

$$\partial_s(\Phi) = \rho^{-1} \tag{2}$$

$$\partial_t(\delta) + \nabla_s \cdot (\delta\mathbf{u}_h) = R_\delta \tag{3}$$

$$\partial_t(\delta T) + \nabla_s \cdot (\delta\mathbf{u}_h T) = R_T \tag{4}$$

$$\partial_t(\delta S) + \nabla_s \cdot (\delta\mathbf{u}_h S) = R_S \tag{5}$$

Here $\mathbf{u}_h = (u, v)$ is the horizontal velocity field, $\Phi = gz$ is the geopotential, δ is the *pressure-thickness* associated with each fluid layer, and T and S are the scalar temperature and salinity variables, respectively.

Coupled Dynamics – Multi-level time integration

We are pursuing a multi-split time-stepping approach for state evolution:

```
function StepOcean(u_h, δ, S, T)
  for (i_4 = 1 to n_4)
    for (i_3 = 1 to n_3)
      for (i_2 = 1 to n_2)
        for (i_1 = 1 to n_1)
          Step (U_h, Δ) for free-surface mode.
        end for
        Step (u_h, δ) for velocity/pressure-thickness.
      end for
      Step (S, T, Q, ...) for tracer-transport.
    end for
    Do vertical re-grid for δ.
    Do vertical remapping for layer state (u_h, S, T).
  end for
```

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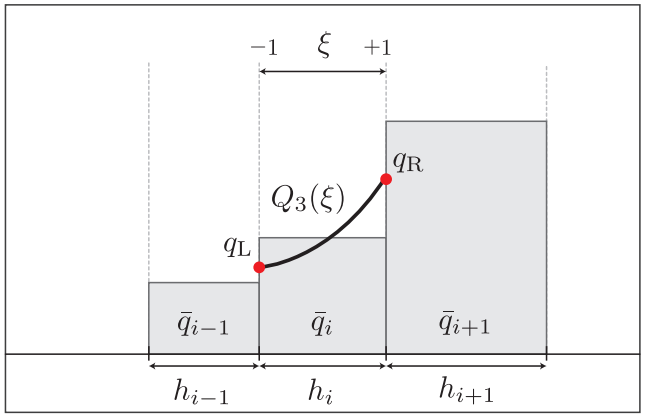
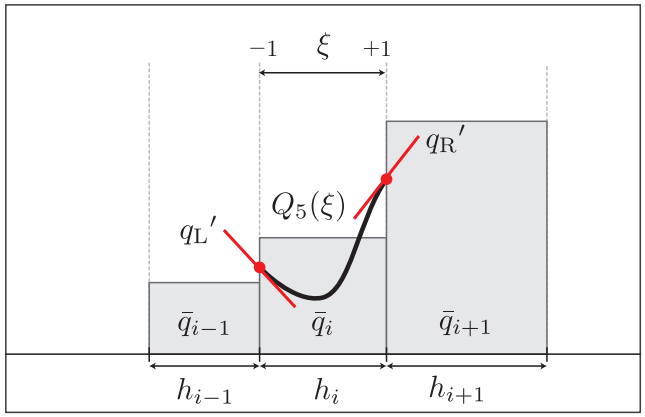
WENO – High-order accurate remapping

High-order accurate vertical 'remapping' can be achieved by fitting high degree polynomials over the column profiles (i.e. layer-mean T, S distributions).





Two popular methods are PPM (*Piecewise Parabolic Method*) and PQM[1] (*Piecewise Quartic Method*).

Conceptually: fit polynomials using both **layer-means** and interpolation of **edge value/slope** information.

1 L. White and A. Adcroft. *A high-order finite volume remapping scheme for nonuniform grids: The piecewise quartic method (PQM)*. Journal of Computational Physics 227(15) (2008) pp. 7394–7422.

WENO – High-order accurate remapping

While the design of high-order accurate PPM/PQM reconstructions is well established, the construction of appropriate high-order **slope-limiters** is not.

A new WENO-type slope-limited PPM/PQM reconstruction[1]

The PPM/PQM methods naturally give rise to both *monotonic* and *natural* polynomial profiles for each grid-cell.

New idea: Form slope-limited profiles $\hat{Q}_l(\xi)$ by taking a convex combination of $\hat{Q}_n(\xi)$ (unlimited) and $\hat{Q}_m(\xi)$ (monotonic) profiles:

$$\hat{Q}_l(\xi) = w_n \hat{Q}_n(\xi) + w_m \hat{Q}_m(\xi) \tag{6}$$

where w_n, w_m are a pair of **non-linear** weights computed by assessing the *local-smoothness* of the data.

(**WENO** – **W**eighted **E**ssentially **N**on-**O**scillatory reconstruction.)

1 D. Engwirda and M. Kelley. *A WENO-type slope-limiter for a family of piecewise polynomial methods*. arXiv preprint arXiv:1606.08188 (2016).

WENO – High-order accurate remapping

The main step in the WENO methodology is the computation of the *blending* weights w_n, w_m. This is done in a way to set $w_n = 1$ when the data is smooth, and $w_m = 1$ when the data is rough:

$$\tilde{w}_n = \frac{10^5}{\max\left(\beta_{i,j}\right)^3}, \qquad \tilde{w}_m = \frac{10^0}{\min\left(\beta_{i,j}\right)^3} \tag{7}$$

Such that the normalised weights effect a blending between the *natural* and *monotonic* profiles:

$$w_n = \frac{\tilde{w}_n}{\tilde{w}_n + \tilde{w}_m}, \qquad w_m = \frac{\tilde{w}_m}{\tilde{w}_n + \tilde{w}_m} \tag{8}$$

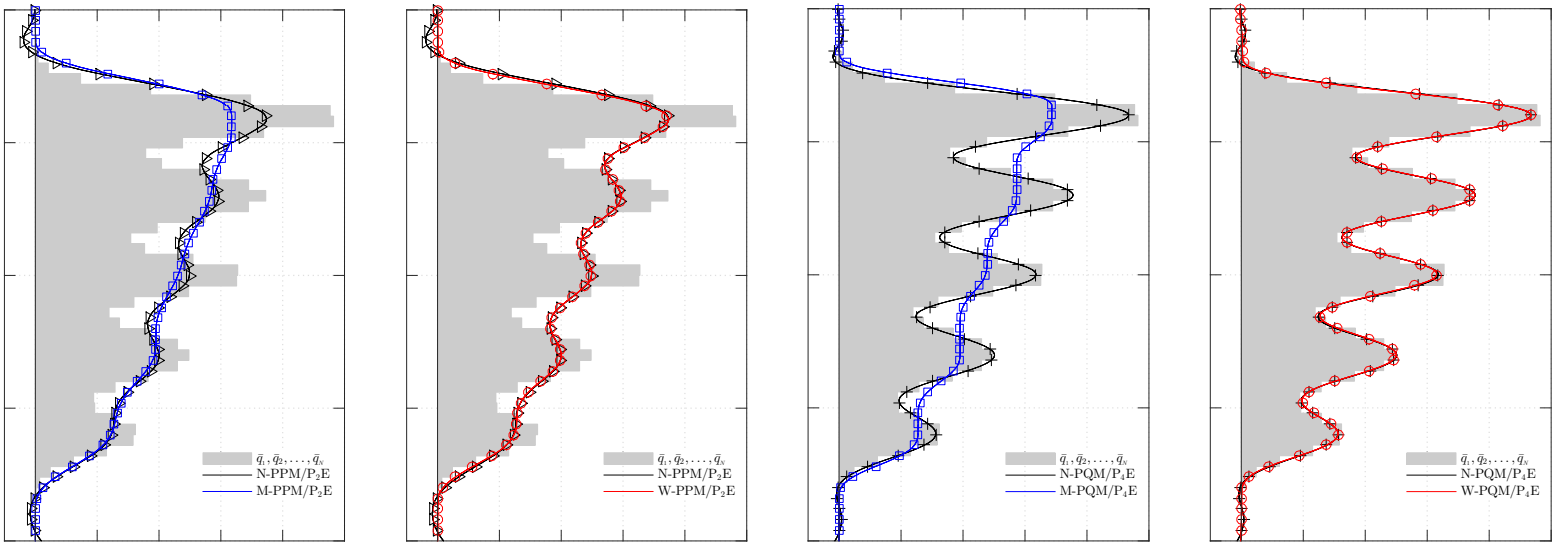
The local *roughness* of the data is computed by examining high-order derivative information:

$$\beta_{i,j} = \sum_{m=1}^{d} \int_{h_i} h_i^{2m-1} \left(\frac{\partial^m Q_{i,j}}{\partial x^m} \right)^2 \mathrm{d}x, \tag{9}$$

WENO considers data to be smooth if it is consistently approximated from the **left** and **right**!

WENO – High-order accurate remapping

Using high-order methods and slope-limiters is worth it!



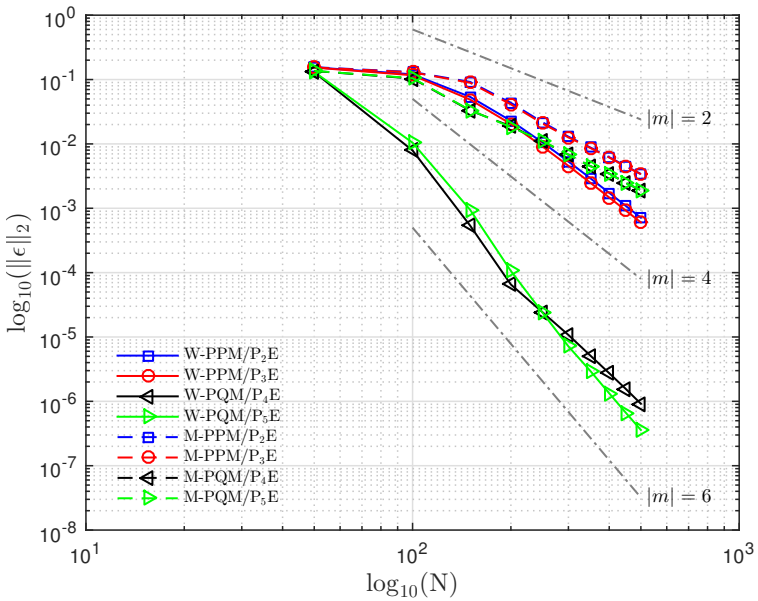
WENO (Weighted Essentially Non-Oscillatory) methods can be thought of as a high-order accurate approach to the problem of 'slope-limiting'.

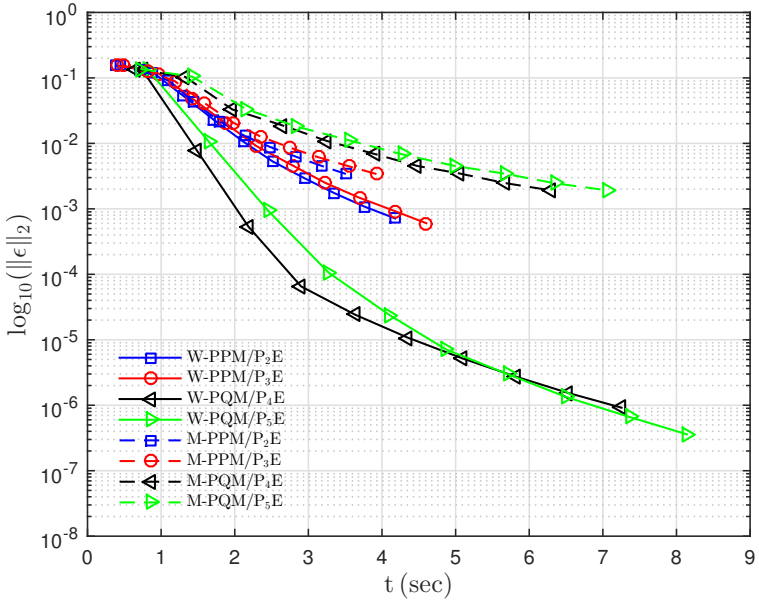
Importantly, they allow for 'smooth' peaks in data to be resolved, while ensuring stability at discontinuities.

Bottom-line is less imposed numerical diffusion.

WENO – High-order accurate remapping

Using high-order methods and slope-limiters is worth it!





WENO-type methods achieve full orders of accuracy/convergence (i.e. 5th-order accurate for WENO-PQM) and lead to better accuracy per computational cost.

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Horizontal Pressure Gradient Force

Considering that: (i) fluid layers are **sloping** and **non-uniform** in thickness, (ii) the density-of-seawater is a **complex**, **nonlinear** function of (S, T, p), and (iii) vertical profiles of S, T are also **nonlinear**, evaluation of the horizontal pressure gradient term $\nabla_s(\Phi)+\rho^{-1}\nabla_s(p)$ along an arbitrary vertical coordinate s is **non-trivial**.

- A small fraction of the vertical force balance can 'contaminate' the horizontal accelerations.
- Such occurrences can cause spurious 'spontaneous motion' from an equilibriated state.

Horizontal Pressure Gradient Force

Adcroft et al.[1] showed that by reinterpreting the PGF in terms of a **contour** integral over the boundary of the velocity control-volumes:

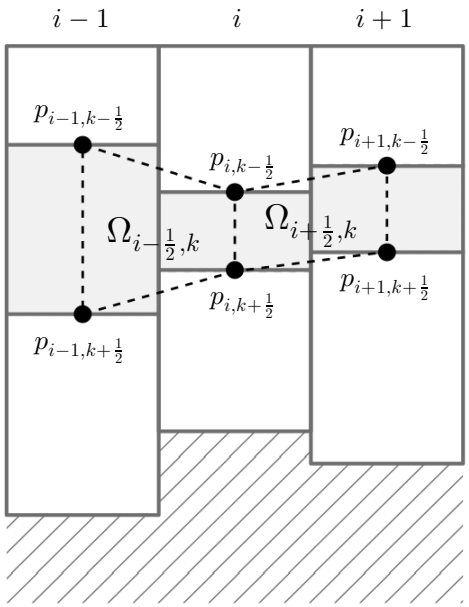
$$\int \nabla_p \Phi \, \mathrm{dV} = \oint_{\partial V} \Phi(x, y, p) \, \mathrm{dC} \tag{10}$$

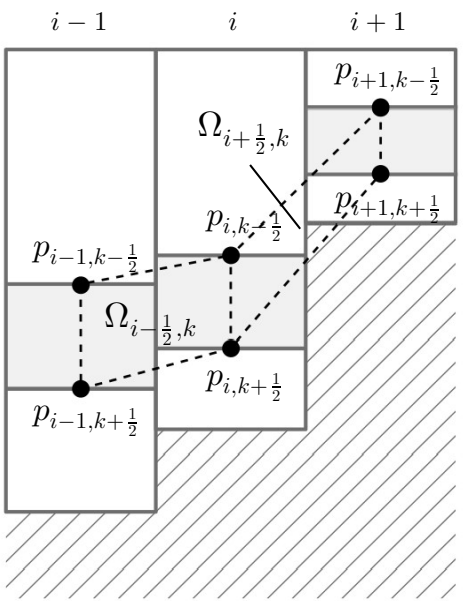
a finite-volume scheme can be developed to **eliminate the hydrostatic and thermobaric errors** arising from **sloping vertical coordinate** systems.

1 A. Adcroft, R. Hallberg and M. Harrison. *A finite volume discretization of the pressure gradient force using analytic integration*. Ocean Modelling 22(3) (2008) pp. 106–113.

Horizontal Pressure Gradient Force

The question is – how best to evaluate the contour integral terms?





Need to account for a number of factors: **pressure-compressibility**, **non-linear** S, T **dependence**, **sloping geometry**.

Horizontal Pressure Gradient Force

In the original semi-analytic approach[1], vertical integrals are computed *exactly* for each control-volume:

$$\begin{aligned}\Phi_t - \Phi_b &= -\int_{p_b}^{p_t} \frac{1}{\rho(S,\theta,p)}\,\mathrm{dp} \\ &= \underbrace{(p_b - p_t)\,A + \lambda \log\left|\frac{P+p_b}{P+p_t}\right|}_{\text{constant across layer thickness}}\end{aligned} \tag{11}$$

But, this result relies on a number of **simplifying assumptions**:

- Fluid density must obey the so-called *Wright* equation-of-state:

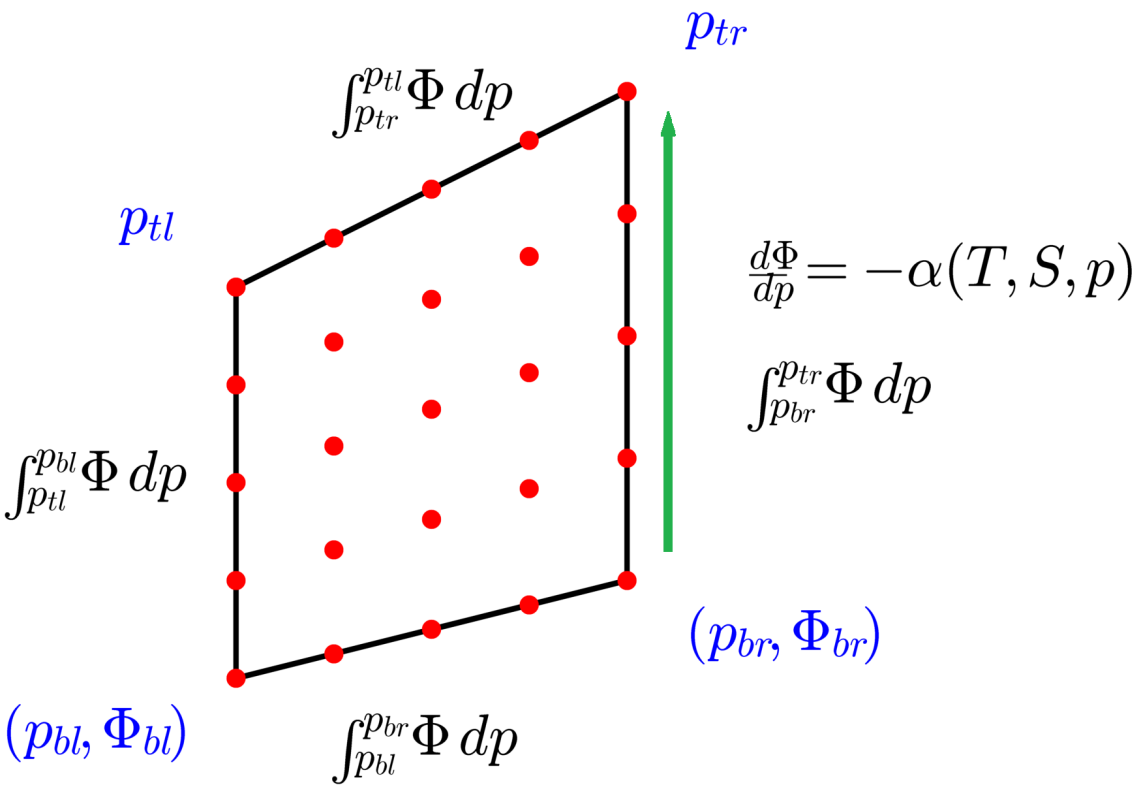
$$\frac{1}{\rho(S,\theta,p)} = A(S,\theta) + \frac{\lambda(S,\theta)}{P(S,\theta)+p} \tag{12}$$

- Through-thickness profiles $S(p)$, $T(p)$ are assumed to be piecewise constant.

1 A. Adcroft, R. Hallberg and M. Harrison. *A finite volume discretization of the pressure gradient force using analytic integration*. Ocean Modelling 22(3) (2008) pp. 106–113.

Horizontal Pressure Gradient Force

To relax such assumptions, our formulation[1] is based on a fully numerical approach (quadrature):



Supports **'black-box'** density functions, higher-order $S(p)$, $T(p)$ profiles.

1 D. Engwirda, M. Kelley and J. Marshall. *A WENO-type slope-limiter for a family of piecewise polynomial methods*. arXiv preprint arXiv:1606.08188 (2016).

Horizontal Pressure Gradient Force

To assess performance, we conducted a set of 'ocean-at-rest' experiments (two-dimensional box, L = 1000km):

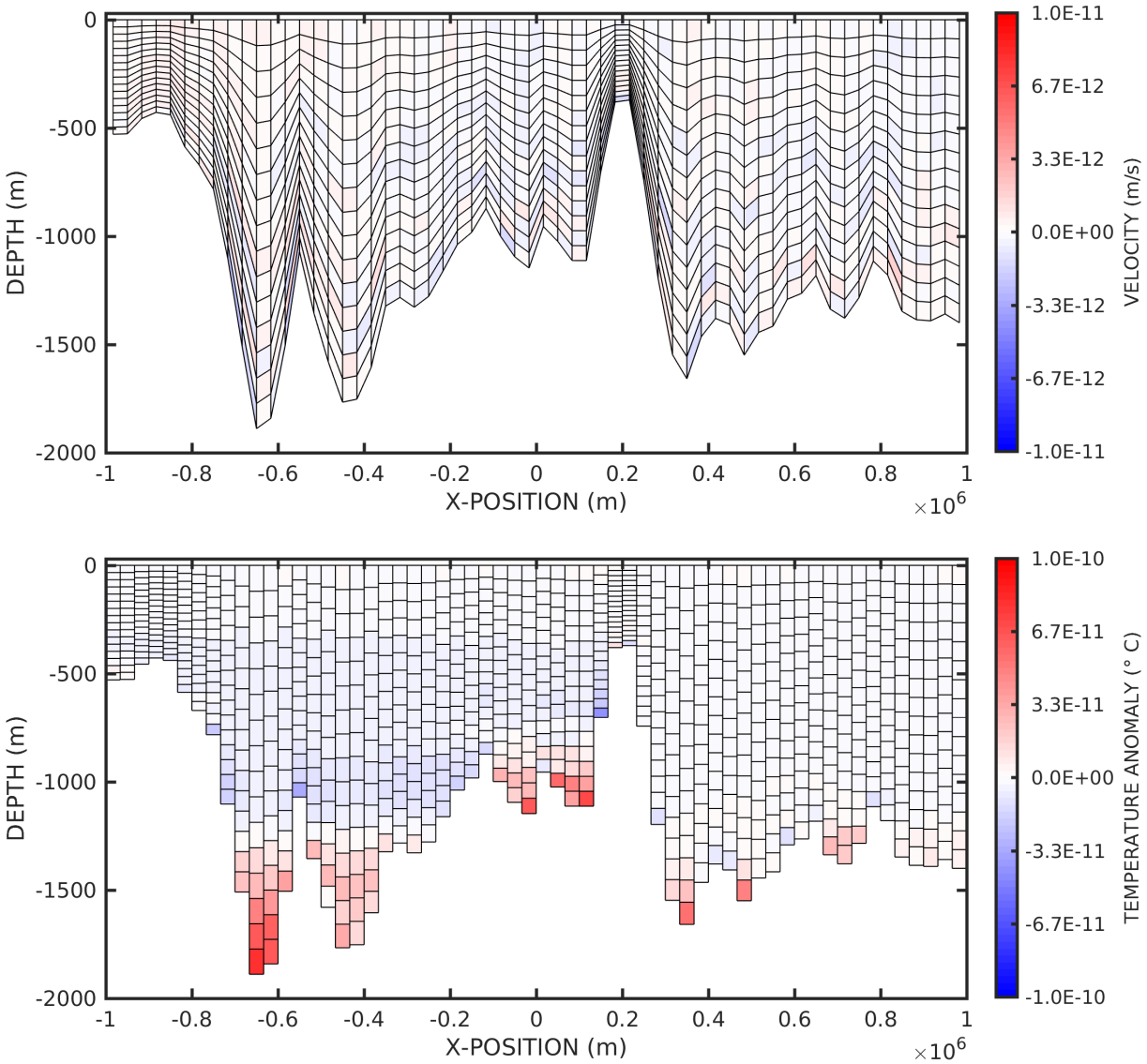
- Prescribed motionless initial conditions.
- Strongly-sloping coordinate surfaces (pure terrain-following layers).
- Prescribed vertical stratification ($S(p)$, $T(p)$ with $\frac{\partial S}{\partial x}, \frac{\partial T}{\partial x} = 0$).

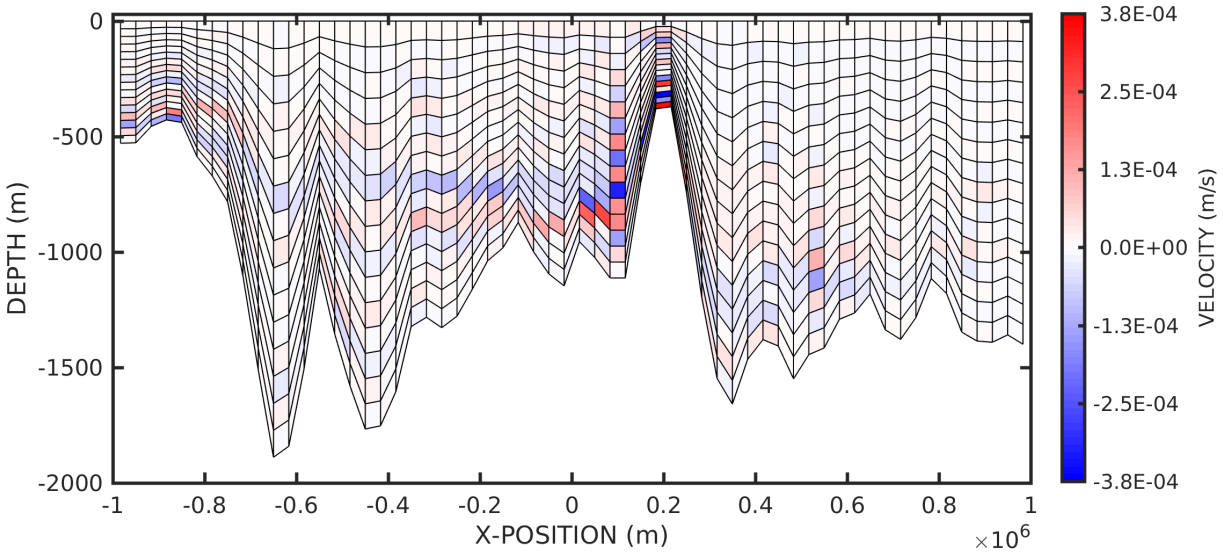
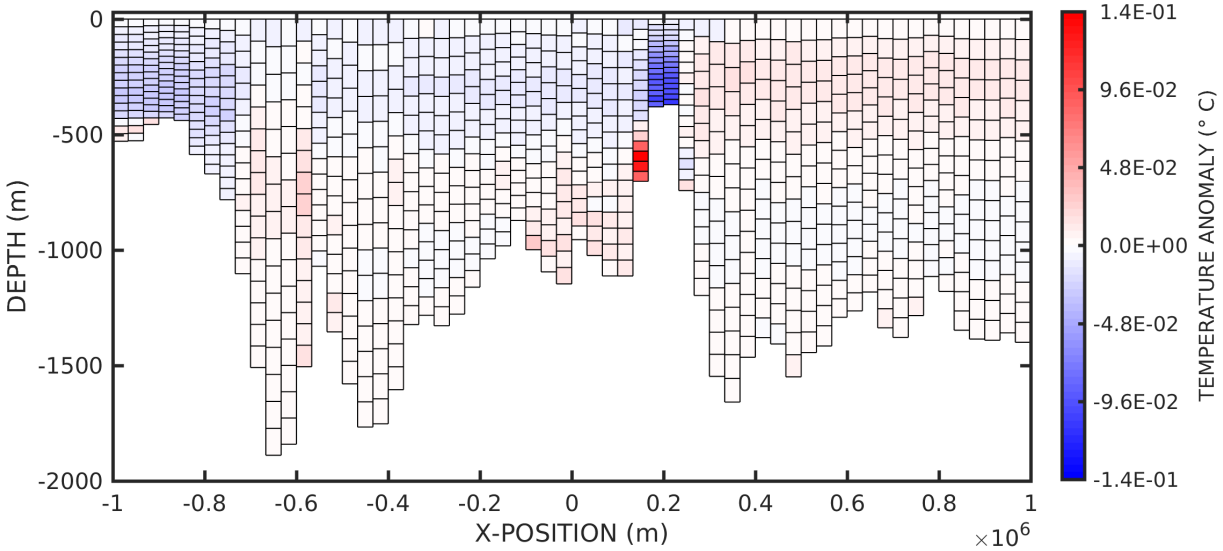
Measure the magnitude of spurious 'drift' away from (nonlinear) equilibrium:

- (Spurious) horizontal velocity components after 90 days.
- Temperature anomaly after 90 days.

Horizontal Pressure Gradient Force

Linear $S(p)$, $T(p)$ profiles ($\Delta S = 20\,{}^{\mathrm{g}}/_{\mathrm{kg}}$, $\Delta T = 25\,^{\circ}\mathrm{C}$), TEOS-10:



Horizontal Pressure Gradient Force

Exponential $S(p)$, $T(p)$ profiles ($\Delta S = 20\,{}^{\mathrm{g}}/_{\mathrm{kg}}$, $\Delta T = 25\,^{\circ}\mathrm{C}$), TEOS-10: