Differentiation Practice II

In each of Questions A21-A24, y is a differentiable function of x. Choose the alternative that is the derivative $\frac{dy}{dx}$

A21. $x^3 - y^3 = 1$

(A) x (B) $3x^2$ (C) $\frac{x^2}{y^2}$ (D) $\frac{3x^2-1}{y^2}$

A22. $x^2 + \cos(x+y) = 0$

(A) $\csc(x+y) - 1$ (B) $\csc(x+y)$ (C) $\frac{x}{\sin(x+y)}$ (D) $\frac{1-\sin x}{\sin y}$

A23. $\sin x - \cos y - 2 = 0$

(A) $-\cot(x)$ (B) $\frac{\cos x}{\sin x}$ (C) $-\csc y \cos x$ (D) $\frac{2-\cos x}{\sin y}$

A24. $3x^2 - 2xy + 5y^2 = 1$

(A) $\frac{3x+y}{x-5y}$ (B) $\frac{y-3x}{5y-x}$ (C) $3x+5y$ (D) $\frac{3x+4y}{x}$

A25. If $x = t^2 + 1$ and $y = 2t^3$, then $\frac{dy}{dx} =$

(A) 3t (B) $6t^2$ (C) $\frac{6t^2}{(t^2+1)^2}$ (D) $\frac{2t^4+6t^2}{(t^2+1)^2}$

A26. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1,2\}$ (B) $\{0,-1,-2\}$ (C) $\{-1,2\}$ (D) $\{0,1,2\}$

A27. If $16\sqrt{x}$, then $f''(4)$ is equal to

(A) -16 (B) -4 (C) -2 (D)-$\frac{1}{2}$

A28. If $f(x) = \ln(x^3)$, then $f''(3)$ is

(A) -$\frac{1}{3}$ (B) -1 (C) -3 (D) 1

A29. If a point moves on the curve $x^2 + y^2 = 25$, then, at (0.5), $\frac{d^2y}{dx^2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) -5 (D) $-\frac{1}{5}$

A30. If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then, when $t = 1, \frac{d^2y}{dx^2}$ is

(A) 1 (B)-1 (C) 3 (D) $\frac{1}{2}$

A31. If $f(x) = 5^x$ and $5^{1.002} \approx 5.016$, which is closest to $f'(1)$?

(A) 0.016 (B) 5.0 (C) 8.0 (D) 32.0

A32. If $y = e^x(x-1)$, then $y"(0)$ equals

(A) -2 (B) -1 (C) 1 (D) e

A33. If $x = e^\theta \cos\theta$ and $y = e^\theta \sin\theta$, then, when $\theta = \frac{\pi}{2}, \frac{dy}{dx}$ is

(A) 1 (B) 0 (C) $e^{\frac{\pi}{2}}$ (D) -1

A34. Given $x = \cos t$ and $y = \cos 2t$, if $\sin t \neq 0$, then $\frac{d^2y}{dx^2}$ is

(A) $4\cos t$ (B) 4 (C) -4 (D) $-4\cos t$

A35. $\lim_{h\to 0} \frac{(1+h)^6-1}{h}$ is

(A) 0 (B) 1 (C) 6 (D) nonexistent

A36. . $\lim_{h\to 0} \frac{\sqrt[3]{8+h}-1}{h}$ is

(A) 0 (B) $\frac{1}{12}$ (C) 1 (D) 192

A37. $\lim_{h\to 0} \frac{\ln(e+h)-1}{h}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e

A38. $\lim_{x\to 0} \frac{\cos x - 1}{x}$ is

(A) -1 (B) 0 (C) 1 (D) ∞

A39. If $f(x) = \begin{cases} \frac{4x^2-4}{x-1} & x \neq 1 \\ 4 & x = 1 \end{cases}$, which of the following statements is (are) true?

I. $\lim_{x\to 1} f(x)$ exists
II. f is continuous at $x = 1$
III. f is differentiable at $x = 1$

(A) Non e (B) I only (C) I and II only (D) I, II, and III

A40. If $g(x) = \begin{cases} x^2 & x \leq 3 \\ 6x - 9 & x > 3 \end{cases}$, which of the following statements is (are) true?

I. $\lim_{x\to 1} g(x)$ exists
II. g is continuous at $x = 3$
III. g is differentiable at $x = 1$

(A) I only (B) II only (C) I and II only (D) I, II, and III

A41. The function $f(x) = x^{2/3}$ on [-8, 8] does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is note defined
(B) $f(x)$ is not continuous on [-8, 8]
(C) $f(x)$ is note defined for $x < 0$
(D) $f'(0)$ does not exist

A42. If $f(x) = 2x^3 - 6x$, at which point on the interval $0 \leq x \leq \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line on that interval ?

(A) 1 (B) $\sqrt{2}$ (C) 0 (D) nowhere

A43. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

(A) -9 (B) -1/9 (C) 1/9 (D) 9

A44. $\lim_{x\to\infty} \frac{e^x}{x^{50}}$ equals

(A) 0 (B) 1 (C) $\frac{1}{50!}$ (D)∞

A45. If $\sin(xy) = x$, then $\frac{dy}{dx}$=

(A) $\frac{\sec(xy)}{x}$ (B) $\frac{\sec(xy)-y}{x}$ (C) $\frac{1+\sec(xy)}{x}$ (D) $\sec(xy) - 1$

A46. $\lim_{x\to 0} \frac{\sin 2x}{x}$ is

(A) 1 (B) 2 (C) ½ (D) 0

A47. $\lim_{x\to 0} \frac{\sin 3x}{\sin 4x}$ is

(A) 1 (B) 4/3 (C)3/4 (D) 0

A48. $\lim_{x\to 0} \frac{1-\cos x}{x}$ is

(A) 0 (B) 1 (C) 2 (D) ∞

A49. $\lim_{x\to 0} \frac{\tan \pi x}{x}$ is

(A) 0 (B) 1 (C) π (D) ∞

A50. $\lim_{x\to\infty} x^2 \sin\frac{1}{x}$

(A) Is 1 (B) is 0 (C) is ∞ (D) oscillates between -1 and 1

A51. The graph in the xy −plane represents by $x = 3 + 2\sin t$ and $y = 2\cos t - 1$, for $\pi \le t \le \pi$, is

(A) a semicircle (B) a circle (C) an ellipse (D) half of an ellipse

A52. . $\lim_{x\to 0} \frac{1-\cos x}{x}$ is

(A) 0 (B) 1/2 (C) 1 (D) 2

In each of Questions A53-A56, a palr of equations that represent a curve parameterically is given. Choose the alternative that is the derivative $\frac{dy}{dx}$

A53. $x = t - \sin t$ and $y = 1 - \cos t$x=

(A) $\frac{\sin t}{1-\cos t}$ (B) $\frac{1-\cos t}{\sin t}$ (C) $\frac{\sin t}{\cos t - 1}$ (D) $\frac{1-x}{y}$

A54. $x = \cos^3 \theta$ and $y = \sin^3 \theta$

(A) $-\cot \theta$ (B) $\cot \theta$ (C) $-\tan \theta$ (D) $-\tan^2 \theta$

A55. $x = 1 - e^{-t}$ and $y = t + e^{-t}$

(A) $\frac{e^{-1}}{1-e^{-t}}$ (B) $e^t + 1$ (C) $e^t - e^{-2t}$ (D) $e^t - 1$

A56. $x = \frac{1}{1-t}$ and $y = 1 - \ln(1-t) \quad (t < 1)$

(A) $\frac{1}{1-t}$ (B) $t - 1$ (D) ½ (D) $\frac{(1-t)^2}{t}$'(x)

A57. The "left half" of the parabola defined by $y = x^2 - 8x + 10$ for $x \leq 4$ is a one-to-one function; therefore, its inverse is also a function, Call that inverse g. Find $g'(3)$

(A) -1/2 (B) -1/6 (C) 1/6 (D) ½

A58. If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, then $(f \cdot g)'(3)$ equals

(A) 0 (B) 1 (C) 3 (D) 6

A59. If $f(x) = \frac{x}{(x-1)^2}$, then the set of x's for which $f'(x)$ exists is

(A) All reals
(B) All reals except $x = 1$ and $x = -1$
(C) All reals except $x = -1$
(D) All reals except $x = 1$

A60. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $-(x+1)^{-2}$
(B) $\frac{-2x}{(x^2+1)^2}$
(C) $\frac{1}{(x+1)^2}$ (D) $\frac{1}{2\sqrt{x}(x+1)}$

A61. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

(A) -1 (B) 1/3 (C) 1 (D)3

A62. If $f(x) = x^3 - 3x^2 + 8x + 5$ and $g(x) = f^{-1}(x)$, then $g'(5) =$

(A) 8 (B) 1/8 (C) 1/53 (D) 53

Part B: Calculator active-directions: Some of the following questions require the use of a graphing calculator

In Questions B1-B8, differentiable functions f and g have the values shown in the table

x	f	f'	g	g'
0	2	1	5	-4
1	3	2	3	-3
2	5	3	1	-2
3	10	4	0	-1

B1. If $A(x) = f(x) + 2g(x)$, then $A'(3) =$

(A) -2 (B) 2 (C) 7 (D) 8

B2. If $B(x) = f(x) \cdot g(x)$, then $B'(2) =$

(A) -20 (B) -7 (C) -6 (D) 13

B3. If $D(x) = \frac{1}{g'(x)}$, then $D'(1) =$

(A) $-\frac{1}{3}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{9}$ (D) $\frac{1}{3}$

B4. If $H(x) = \sqrt{f(x)}$, then $H'^{(x)} =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) $\frac{2}{\sqrt{10}}$ (D) $4\sqrt{10}$

B5. If $K(x) = \frac{f(x)}{g(x)}$, then $K'(0) =$