

1 **The Effects of Surface Heterogeneity Scale on the Flux Imbalance**

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12 **Key Points:**

- 13
- 14 • The ratio of boundary-layer height to the Obukhov length (z_i/L), the integral length scale
15 of vertical velocity (l_w), the mean horizontal velocity (U), and the average interval (T)
control the flux imbalance (I);
 - 16 • Based on LES results, a diagnostic equation for the flux imbalance is proposed as $I = 1 -$
17 $[az/z_i + b][-K \times z_i/L \times l_w/UT + C]$, where a , b , K , and C are empirical constants and b
18 should in theory equal to unity.
 - 19 • Compared to the other models, this model is derived from the cospectrum and hence is
20 more physically based. Moreover, the relations between the flux imbalance and various
21 factors reported in the literature can be explained by this model.
22

23 **Abstract** It is well known that the available energy (i.e., the net radiation minus the ground heat
24 flux) is often 10% ~ 30% larger than the sum of turbulent fluxes measured by the eddy-
25 covariance method. Although field observations and previous large-eddy simulation (LES)
26 studies have shown that surface heterogeneity can induce flux imbalance, the relationship
27 between the flux imbalance magnitude and the surface heterogeneity scale remains
28 underexplored. In this paper, we examine the flux imbalance over landscapes characterized by
29 different surface heterogeneity scales in a dry convective boundary layer. Results show that the
30 flux imbalance initially increases with increasing surface heterogeneity scale, reaches its peak
31 value when the surface heterogeneity scale is on the order of the convective boundary-layer
32 height (z_i), and then slightly decreases. Based on LES results, we propose a conceptual model to
33 explain how flux imbalance is influenced by surface heterogeneity. We argue that the ratio of
34 boundary-layer height to the Obukhov length (z_i/L), the integral length scale of vertical velocity
35 (l_w), the mean horizontal velocity (U), and the average interval (T) control the flux imbalance.
36 Among these four variables, l_w determines the size of turbulent coherent structures (i.e., large
37 eddies); whereas z_i/L affects the form of these large eddies. Meanwhile, the U and T determine
38 the number of these large eddies that can be sampled by the eddy-covariance sensors in a finite
39 averaging period. This finding indicates that it may be possible to diagnose the flux imbalance
40 using these four variables.

41 **Keywords** Convective boundary layer • Flux imbalance • Large eddy simulation • Surface
42 heterogeneity scale

43 1 Introduction

44 The eddy-covariance (EC) method is one of the most common methods for measuring
45 turbulent exchanges between the biosphere and the atmosphere. The fluxes of heat, water vapor
46 and CO₂ measured at EC sites around the world (e.g., FLUXNET) are widely used for
47 developing and validating land surface schemes in climate models (Williams et al., 2009), and
48 for climatological studies (Jung et al., 2010). However, a long-standing problem with the EC
49 method is observed in which, the sum of the sensible and latent heat fluxes by EC is smaller than
50 the difference between the net radiation and ground heat flux (Aubinet et al., 2012; Foken, 2008;
51 Leuning et al. 2012; Oncley et al., 2007; Twine et al., 2000; Wilson et al., 2002; Xu et al., 2017).
52 This systematic bias in the EC method is called the surface energy imbalance or non-closure
53 problem and has been one of the biggest challenges for using EC measurements.

54 Potential reasons for this energy imbalance include mismatch in the footprints of
55 radiation and turbulent flux measurements, measurement/computation errors, significant
56 advective fluxes, and inadequate sampling of large-scale, low-frequency turbulent eddies, as
57 reviewed elsewhere (Mahrt, 1998; Twine et al., 2000; Foken et al., 2006; Foken, 2008; Wang et
58 al., 2009; Foken et al., 2011; Leuning et al. 2012; Wohlfahrt and Widmoser, 2013). Among these
59 potential causes, the inadequate sampling of large-scale turbulent eddies has been increasingly
60 acknowledged as one of the leading contributors to the surface energy imbalance (Foken et al.,
61 2011).

62 Over homogeneous terrain, the largest turbulent eddy scales with the boundary-layer
63 height (Wyngaard, 1985; Stull, 1988). Over heterogeneous terrain, secondary circulations can be
64 further induced by the landscape-level heterogeneity around EC sites (Foken et al., 2006). These
65 large-scale turbulent eddies and secondary circulations may not be adequately sampled by an EC
66 tower in a finite averaging period (e.g., 30 mins); hence, an imbalance occurs. This phenomenon

67 has been confirmed by many observational (Panin et al., 1998; Stoy et al., 2013; Eder et al.,
68 2015a; Gao et al., 2017; Xu et al., 2017) and large-eddy simulation (LES) studies (Kanda et al.,
69 2004; Inagaki et al., 2006; Steinfeld et al., 2007; Huang et al., 2008; Eder et al., 2015b;
70 Schalkwijk et al., 2016; De Roo and Mauder, 2018; Zhou et al., 2018).

71 Over the past decade, the understanding of flux imbalance has been significantly
72 advanced and many factors were found to be related to the flux imbalance (see Table 1).
73 However, it remains elusive as to (i) how the flux imbalance is influenced by surface
74 heterogeneity and (ii) what the relationship between the flux imbalance magnitude and the
75 surface heterogeneity scale is. For example, although previous studies have shown that surface
76 heterogeneity is a major factor causing flux imbalance (Zhou et al., 2018) and various energy
77 balance closure parametrization schemes have been proposed (Panin et al., 1998; Huang et al.,
78 2008; Panin and Bernhofer, 2008), the key flow and surface variables controlling the flux
79 imbalance magnitude remain debated (Eder et al., 2014). In addition, while field observations
80 (Stoy et al., 2013; Xu et al., 2017) have shown that the flux imbalance magnitude generally
81 increases as the surface heterogeneity becomes stronger, large variations occur in such relations.
82 Addressing these two questions frames the scope of our study.

83 The paper is structured as follows: Section 2 describes the method; Section 3 provides
84 details on the LES model and the numerical experiments. The results are described in Section 4
85 and discussed in Section 5. Finally, Section 6 concludes the paper.

86

87 **Table 1** A summary of factors affecting the flux imbalance (I)

Fators	Relations to I	References
Averaging time (T)	The smaller T , the larger I	Finnigan et al. (2003); Charuchittipan et al. (2014); Kanda et al. (2004); Chen et al. (2006); Steinfeld et al. (2007); Schalkwijk et al. (2016); Zhou et al. (2018)
Height (z)	The larger z , the larger I	Kanda et al. (2004); Chen et al. (2006); Steinfeld et al. (2007); Huang et al (2008); Schalkwijk et al. (2016); Zhou et al. (2018)
Horizontal mean velocity (U)	The smaller U , the larger I	Lee (1998); Stoy et al. (2013); Li et al. (2005); Franssen et al. (2010); Kanda et al. (2004); Chen et al. (2006); Steinfeld et al. (2007); Huang et al. (2008); Schalkwijk et al. (2016); Zhou et al. (2018)
The phase difference	The larger the phase difference between ground heat flux and net radiation, the larger I ; The larger the phase difference between vertical velocity and the scalar, the larger I	Gao et al. (2010); Gao et al. (2017)
Surface heterogeneity	The stronger surface heterogeneity, the larger I	Panin et al. (1998); Stoy et al. (2013); Xu et al. (2017); Zhou et al. (2018); Inagaki et al. (2006)
u_*/w_*	The smaller u_*/w_* , the larger I	Huang et al. (2008); Zhou et al. (2018)
z_i/L	The larger $-z_i/L$, the larger I	Schalkwijk et al. (2016)
z/L	The larger $-z/L$, the larger I	Franssen et al. (2010); McGloin et al. (2018)
Turbulent intensity	The smaller turbulent intensity, the larger I ; the smaller turbulent kinetic energy, the larger I	Zuo et al. (2012); Zhou et al. (2018)

88

89 **2 Method**

90 2.1 Flux imbalance over heterogeneous surfaces

91 We consider a variable φ with the spatial mean as $[\varphi]$ and the temporal mean as $\overline{\varphi}$.
 92 Fluctuations from the spatial and temporal means are denoted as φ'' and φ' , respectively. The
 93 flux imbalance magnitude (I) is defined as the difference between “true flux” and the turbulent
 94 flux, which can be expressed as follows:

$$I = \frac{TF - TU}{TF}, \quad (1)$$

95 where TU denotes the turbulent flux measured by the EC and TF represents the “true flux”. Note
 96 that here I represents the flux imbalance magnitude as TU is often smaller than TF . This is
 97 slightly different from the flux imbalance defined in previous studies (Kanda et al., 2004; Inagaki
 98 et al., 2006; Steinfeld et al., 2007; Huang et al., 2008; Schalkwijk et al., 2016; Zhou et al., 2018),
 99 which is often ($-I$).

100 In Finnigan et al. (2003), the authors introduced the assumptions and limitations of the
 101 EC method using a control volume approach. In this method, the surface heat flux is treated as
 102 the “true flux”. The spatiotemporally averaged budget equation for the potential temperature
 103 over the cubic control volume can be expressed as follows:

$$[\overline{HFX}] = [\overline{w'\theta'}] + \sum_1^4 [\overline{v'\theta'}] + [\overline{w\theta}] + \sum_1^4 [\overline{v\theta}] + \left[\int \frac{\partial \theta}{\partial t} dz \right], \quad (2)$$

104 where HFX denotes the surface heat flux (“true flux”); w and θ indicate the vertical velocity and
 105 potential temperature, respectively; v represents the velocity vector perpendicular to the lateral
 106 faces and $\overline{w'\theta'}$ represents the temporally-averaged vertical turbulent heat flux as measured by
 107 the EC method at a single location. Clearly, the other terms on the right-hand side of Eq. (2)
 108 denote the items causing the flux imbalance: the horizontal flux divergence, the vertical mean
 109 advection, the horizontal mean advection, and the storage of θ within the control volume. In this
 110 study, we do not separate the horizontal flux divergence and the vertical and horizontal mean
 111 advection terms. The sum of these three terms (the horizontal flux divergence and the vertical
 112 and horizontal mean advection) is referred to as the “advection” term in this study. The fraction
 113 of “advection” term to the true flux is referred to as the “advection fraction”.

114 The domain averaged flux imbalance magnitude over homogeneous surfaces can thus be
 115 calculated as follows:

$$[I] = \left[\frac{[\overline{HFX}] - \overline{w'\theta'}}{[\overline{HFX}]} \right], \quad (3)$$

116 where $[I]$ represents the “mean” difference between the temporal flux (i.e., the flux calculated
 117 from the EC method) and the true flux. It should be pointed out that in some previous studies
 118 (Kanda et al., 2004; Inagaki et al., 2006; Steinfeld et al., 2007; Huang et al., 2008; Schalkwijk et
 119 al., 2016; Zhou et al., 2018), the spatiotemporally averaged vertical heat flux ($[\overline{w\theta}]$) at a given
 120 height is used as the “true flux”. This method is often used over homogeneous surfaces (Kanda et

121 al., 2004; Steinfeld et al., 2007; Huang et al., 2008; Schalkwijk et al., 2016; Zhou et al., 2018)
 122 and 1-D heterogeneous surfaces (Inagaki et al., 2006; Zhou et al., 2018). In this study, however,
 123 we use the surface flux as the true flux.

124 2.2 The surface heterogeneity scale

125 Many previous LES studies focused on idealized heterogeneity, such as striped-like and
 126 chessboard-like heterogeneities with uniform sizes (Chen and Avissar, 1994; Avissar and
 127 Schmidt, 1998). For these idealized heterogeneity patterns, the characteristic length scale can be
 128 relatively easily defined. For example, the width of the strip can be used to indicate the length
 129 scale of surface heterogeneity. Recently, more complex and/or realistic landscape patterns based
 130 on remote sensing have been used (Albertson et al., 2001; Kustas and Albertson, 2003; Bertoldi
 131 et al., 2007; Huang and Margulis, 2009; Liu et al., 2017). To characterize the length scale of
 132 surface heterogeneity (L_p) over complex landscape patterns, Bou-Zeid et al., (2007) (hereafter
 133 B07) proposed to define L_p as follows:

$$L_p = \int_0^{L_D} \left[1 - \frac{D(r)}{\max(D)} \right] dr, \quad (4)$$

134 where L_D is the streamwise length of the domain and $D(r)$ is the structure function of the
 135 analyzed surface characteristics (e.g., momentum roughness length) with distance r along the
 136 streamwise direction. On the other hand, Huang and Margulis (2009) (hereafter HM09) used a
 137 normalized exponential variogram model to define the characteristic length scale:

$$L_p = -\frac{\ln \gamma}{3d}, \quad (5)$$

138 where the d is the distance matrix representing the distance between any two points and γ is the
 139 variogram. The details about the two methods can be found in B07 and HM09.

140 2.3 A cospectral model

141 In this study, we employ the cospectrum, which describes the distribution of flux among
 142 scales, to interpret the flux imbalance. Figure 1a presents a schematic illustrating the cospectrum
 143 of heat flux ($F_{w\theta}$), where k is the wavenumber and k_p is the wavenumber at which $F_{w\theta}$ reaches its
 144 peak (hereafter the peak wavenumber). Within the confines of the cospectrum, we assume that
 145 the EC method can only sample eddies that are larger (smaller) than a critical wavenumber
 146 (wavelength), which is referred to as k_{ec} (Fig. 1a). This implicitly assumes that the large eddies
 147 are responsible for the flux imbalance, which can be calculated as follows:

$$[I] = 1 - \frac{\int_{k_{ec}}^{\infty} F_{w\theta}(k, z) dk}{HFX} = 1 - \frac{\int_0^{\infty} F_{w\theta}(k, z) dk \left(1 - \frac{\int_0^{k_{ec}} F_{w\theta}(k, z) dk}{\int_0^{\infty} F_{w\theta}(k, z) dk} \right)}{HFX}. \quad (6)$$

148 The above expression can be further simplified as follows,

$$[I] = 1 - f_1 f_2. \quad (7)$$

$$f_1 = \frac{\int_0^{\infty} F_{w\theta}(k, z) dk}{HFX}. \quad (8)$$

$$f_2 = 1 - \frac{\int_0^{k_{ec}} F_{w\theta}(k, z) dk}{\int_0^{\infty} F_{w\theta}(k, z) dk}. \quad (9)$$

149 In Eq. (7), f_1 is the ratio of the “flux at a certain height” to the surface flux and its
 150 deviation from unity represents the effect of storage/advection terms in Eq. (2). The term f_2
 151 represents the role of large eddies in causing flux imbalance at any given height z . From Eq. (7)
 152 it can be seen that the flux imbalance magnitude increases with decreasing f_1 and decreasing f_2 .

153 Different functional forms have been used for the cospectrum $F_{w\theta}$ (Kaimal and Finnigan,
 154 1994; Massman and Lee, 2002; Lee et al., 2005; Katul et al., 2013). In this paper, we use the
 155 cospectrum model of Lee et al. (2005) for illustration purpose, as follows:

$$F_{w\theta}(k) = \frac{A}{k_p \left[1 + m \left(\frac{k}{k_p} \right)^{2\mu} \right]^{\frac{1}{2\mu} \frac{m+1}{m}}}, \quad (10)$$

156 where A is a normalization parameter, m and μ are the (inertial subrange) slope parameter and
 157 broadness parameter, respectively. When $m = 3/4$, Eq. (10) can produce the $-7/3$ power law of
 158 $F_{w\theta}$ in the inertial subrange (Lumley, 1967; Li and Katul, 2017). When $\mu = 0.5$, Eq. (10) can
 159 reproduce the observed cospectrum from the famous Kansas experiment (Kaimal et al., 1976),
 160 which is commonly used as the standard in the cospectral correction of EC observations (Moore,
 161 1986). Substituting Eq. (10) into Eq. (6), the f_2 term can be expressed as:

$$f_2 = \frac{\Gamma\left(\frac{1}{2\mu}, \frac{m+1}{2\mu} \left(\frac{k_{ec}}{k_p}\right)^{2\mu}\right)}{\Gamma\left(\frac{1}{2\mu}, 0\right)}, \quad (11)$$

162 where Γ is the incomplete Gamma function. The detailed derivation of Eq. (11) is presented in
 163 Appendix A. Because m and μ are positive and $\Gamma(s, x)$ decreases with increasing x , Eq. (11)
 164 indicates that f_2 decreases with increasing k_{ec}/k_p . Similar results as Eq. (11) are obtained using
 165 other cospectral models as shown in Appendix B but the relation between f_2 and k_{ec}/k_p strongly
 166 depends on the assumed cospectral model. As such, we will not use the exact relation between f_2
 167 and k_{ec}/k_p given by the Lee et al. (2005) model (namely, Eq. 11) in our calculation. Instead, the
 168 Eq. (11) is meant to provide a qualitative description of the relation between f_2 and k_{ec}/k_p (i.e., f_2
 169 is expected to decrease with increasing k_{ec}/k_p). In Sect. 4.2, we will first explore the behavior of
 170 f_1 , which will be calculated from Eq. (8), and then explore the relation between f_2 , which will be
 171 inferred from $(1-[I])/f_1$, and k_{ec}/k_p .

172 Now we detail how k_{ec}/k_p is estimated (or approximated). Based on the sampling theory,
 173 the larger the eddy moving velocity (say, represented by u), the more eddies can be sampled by
 174 the EC in a finite averaging period (T) and hence the smaller the flux imbalance. Therefore, k_{ec}
 175 may be represented by $1/(uT)$. A reasonable estimate for u is the mean horizontal velocity U by
 176 invoking the Taylor’s hypothesis. However, given the limitation of Taylor’s hypothesis in
 177 constructing very large-scale structures in turbulent boundary layers (Dennis and Nickels, 2008),
 178 we will also examine other velocity scales such as the convective velocity w_* , which is defined
 179 as follows:

$$w_* = \left(\frac{g}{\rho} z_i \frac{H}{c_p \theta_0} \right)^{\frac{1}{3}}, \quad (12)$$

180 where g is the gravitational acceleration, θ_0 is the reference potential temperature, c_p is the
 181 specific heat of air, and z_i is the boundary-layer height. In addition, we will consider the friction
 182 velocity $u_* = \sqrt{\tau/\rho}$, where τ is the surface drag and ρ is the air density, as well as the square
 183 root of turbulent kinetic energy (TKE) ($\bar{e} = \sqrt{TKE} = \sqrt{0.5(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)}$, where σ_u^2 , σ_v^2 , and
 184 σ_w^2 are the variances of the three wind velocity components). Note that w_* and u_* have been used
 185 in previous studies to characterize the flux imbalance magnitude (Huang et al., 2008).

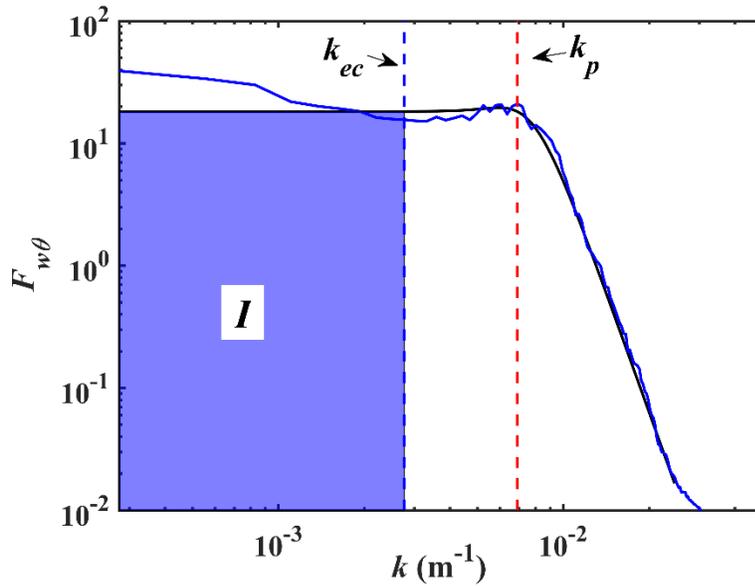
186 In addition to k_{ec} , we also need an estimate of k_p , namely, the peak wavenumber. The
 187 peak wavenumber k_p can be related to the inverse of the integral length scale or $1/l$ (Kaimal and
 188 Finnigan, 1994), which is an estimate of the size of organized turbulent structures in the
 189 turbulent boundary layer. It is defined based on the autocorrelation function R as follows:

$$l = \int_0^{\infty} R(r) dr, \quad (13)$$

190 where r is the displacement in space. In this study, l is calculated using the autocorrelation
 191 function of vertical velocity (l_w) or potential temperature (l_θ) with an integration until the first
 192 zero of $R(r)$ (Lenschow and Stankov, 1986).

193 In a neutrally stratified atmospheric surface layer over homogenous surfaces, the
 194 integrate length scale is on the order of z (Kaimal et al., 1976), yielding that $[l]$ increases with
 195 $z/(uT)$. This result is in agreement with previous results showing that the flux imbalance
 196 magnitude increases with increasing height in the surface layer, decreasing mean horizontal
 197 velocity (when u is represented by the mean horizontal velocity), and shorter averaging period
 198 over homogeneous surfaces (Kanda et al., 2004; Steinfeld et al., 2007; Schalkwijk et al., 2016;
 199 Zhou et al., 2018). This agreement shows that the result from the cospectral model is reasonable
 200 (at least over homogeneous surfaces).

201 It is pointed out again that the relation between f_2 and k_{ec}/k_p is strongly dependent on the
 202 assumed cospectral shape. That is, f_2 is not only a function of k_{ec}/k_p as Eq. (11) alludes to. While
 203 $l/(uT)$ might provide a good approximation to k_{ec}/k_p , other factors that can affect the cospectral
 204 shape (but not explicitly considered by the Lee et al. (2005) cospectral model) need to be further
 205 considered. These factors include surface heterogeneity and atmospheric stability, the latter of
 206 which can be indicated by z_i/L , where L is the Obukhov length. We choose z_i/L because previous
 207 studies (Lee et al, 2005; Moeng and Sullivan, 1994; Khanna and Brasseur, 1998; Schalkwijk et
 208 al., 2016) have found that z_i/L strongly affects large-scale turbulent coherent structures and thus
 209 the low wavenumber regime of the cospetra (Kaimal and Finnigan, 1994). For example, when
 210 $z_i/|L|$ is larger than about 25, cell structures form with polygonal patterns and no noticeable
 211 alignment. When $z_i/|L|$ is larger than 5 and smaller than 25, roll structures formed aligned with
 212 the wind (Lee et al, 2005). Moreover, previous studies have also found a correlation between the
 213 flux imbalance and z_i/L (Schalkwijk et al., 2016; Huang et al., 2008). Therefore, in this study, we
 214 use z_i/L to represent the effects of atmospheric stability on turbulent coherent structures and
 215 hence on the flux imbalance.



216

217 **Figure. 1** A conceptual model for flux imbalance from the cospectral perspective. The blue curve is the
 218 cospectrum calculated based on the LES results of B2000 (see Sect. 3.2) and the black curve is the fitted
 219 cospectrum based on Eq. (10) with $k_p=0.007$, $m=0.18$ and $\mu=5.5$. The k is the wavenumber and $F_{w\theta}$ is the
 220 cospectrum between w and θ . I is the flux imbalance. The k_p and k_{ec} are the peak wavenumber and critical
 221 wavenumber for EC, respectively.

222 3 Experimental design

223 3.1 Model description and configuration

224 It is pointed out again that the relation between f_2 and k_{ec}/k_p is strongly dependent on the
 225 assumed cospectral shape. That is, f_2 is not only a function of k_{ec}/k_p as Eq. (11) alludes to. While
 226 $l/(uT)$ might provide a good approximation to k_{ec}/k_p , other factors that can affect the cospectral
 227 shape (but not explicitly considered by the Lee et al. (2005) cospectral model) need to be further
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 236 flux imbalance and z_i/L (Schalkwijk et al., 2016; Huang et al., 2008). Therefore, in this study, we
 237 use z_i/L to represent the effects of atmospheric stability on turbulent coherent structures and
 238 hence on the flux imbalance.

239 We used the Weather Research and Forecasting (WRF) model version 3.9, which has the
 240 LES capability and has been widely used to investigate convective boundary layers over
 241 homogeneous and heterogeneous surfaces (Moeng et al., 2007; Patton et al., 2005; Talbot et al.,
 242 2012; Zhu et al., 2016). Following Zhu et al., (2016), the original WRF-LES model was

243 modified so that the surface temperature, instead of the surface heat flux, can be prescribed as the
244 surface boundary condition.

245 Except for the surface layer scheme, other physical schemes in the WRF-LES model such
246 as microphysics and radiation are all turned off. In the surface layer scheme, Monin-Obukhov
247 similarity theory is used to calculate sensible heat flux from the prescribed surface temperature
248 and the simulated air temperature. Periodic boundary conditions are used in our simulations. In
249 addition, we use the default WRF numerical discretization options (i.e., a fifth-order scheme for
250 advection in the horizontal direction, a third-order scheme for advection in the vertical direction
251 and a third-order Runge-Kutta scheme for the time integration). For the subgrid-scale turbulence
252 parameterization, the 1.5-order turbulent kinetic energy-based closure scheme is used. The
253 subgrid-scale flux is added to the turbulent flux.

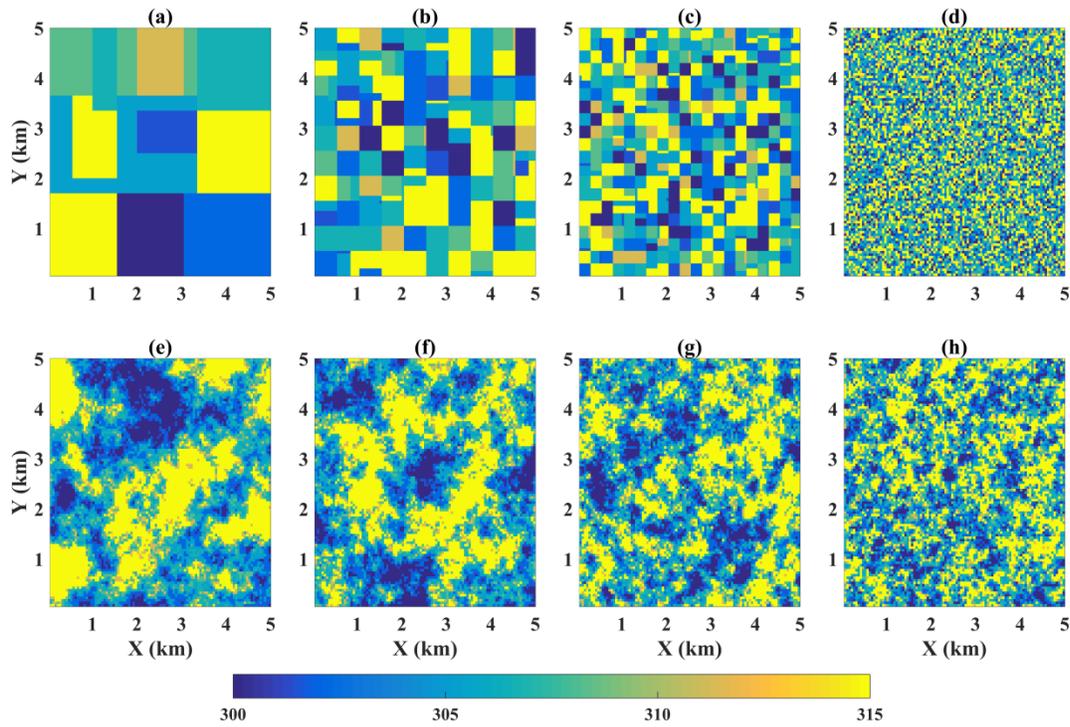
254 The simulation domain is $5 \text{ km} \times 5 \text{ km} \times 2 \text{ km}$ in the x , y and z directions, respectively,
255 and the number of grid points is $100 \times 100 \times 100$. Hence, the resolution in x - and y - directions is
256 50 m and ranges from $6 \text{ m} \sim 20 \text{ m}$ in the z -direction (Talbot et al., 2012). The time step is 0.25 s .
257 The model is initialized with an idealized neutral boundary-layer profile, in which the potential
258 temperature is 298 K below 850 m , with a strong inversion layer of a potential temperature surge
259 of 60 K km^{-1} from 850 to 1050 m . The potential temperature gradient is 3 K km^{-1} above 1050 m .
260 All cases had the same initial atmospheric conditions. The geostrophic wind is set to be zero in
261 all cases.

262 3.2 Numerical Experiments

263 In this study, we simulate a dry atmosphere over a heterogeneous surface composed of
264 random configurations of patches with different surface temperatures and momentum roughness
265 lengths. We design two cases to analyze the effects of different heterogeneity scales on the flux
266 imbalance. The key information is summarized in Table 2.

267 Similar to B07, a baseline surface configuration is created manually with relatively large
268 patches (Fig. 2a). Subsequently, the baseline configuration is changed to create surfaces with
269 different heterogeneity length scales. With different methods, two types of simulations are
270 designed. The cases denoted by ‘B’ used the method from B07 (e.g., Eq. 6) and cases denoted by
271 ‘H’ used the method from HM09 (e.g., Eq. 7). Due to the use of the same baseline configuration,
272 the probability density functions (pdfs) of surface properties are identical in the two types of
273 simulations. For each type, four simulations are performed with different heterogeneity length
274 scales (e.g., 2000 m , 1200 m , 550 m and 240 m). These scales are chosen to give a broad range
275 of surface heterogeneity scales: one simulation with a heterogeneity scale larger than the
276 boundary-layer height (2000 m), one simulation with a heterogeneity scale on the order of
277 boundary-layer height (1200 m), and two simulations with heterogeneity scales smaller than the
278 boundary-layer height (550 m and 240 m). The spatial patterns of surface temperature in the
279 above mentioned simulations are shown in Fig. 2. Note that the statistical properties (i.e., the
280 pdfs) in all cases are same.

281 For all simulations, a 2-hour spin-up is used (Patton et al., 2005). The temporal statistics
282 are calculated for all grid points, i.e., all the grids in the domain are virtual towers.



283

284 **Figure. 2** Spatial patterns of surface temperature in B cases (a to d) and H cases (e to h).

285

286 **Table 2.** Summary of the experiments along with spatially and temporally averaged ABL characteristics in
 287 case B and H. The z_i represents the boundary-layer height and the u_* and w_* indicate the friction velocity and
 288 convective velocity, respectively.

Case Name	Heterogeneity scales (m)	z_i (m)	u_* (m/s)	w_* (m/s)
B2000	2000	1215	0.37	2.64
B1200	1200	1211	0.36	2.74
B550	550	1168	0.40	2.73
B240	240	1154	0.43	2.62
H2000	2000	1254	0.42	2.91
H1200	1200	1249	0.40	2.90
H550	550	1210	0.43	2.80
H240	240	1228	0.43	2.80

289

290 **4 Results**

291 4.1 The effects of surface heterogeneity scale on flux imbalance magnitude

292 4.1.1 The simulated ABL structures

293 As shown in Fig. 2, the patterns of surface temperature (and roughness length, not
294 shown) are significantly different between B cases and H cases. As a result, the vertical profiles
295 of momentum and scalars under convective conditions are also different, which can be clearly
296 seen from Fig. 3 for B2000 and H2000. The horizontal slice is at 2 m above ground. Clearly,
297 “plumes” can be observed emanating from places where the surface temperature is higher.
298 However, the spatial distributions of these “plumes” are significantly different because of the
299 different spatial configurations of surface temperature. As the heterogeneity scale increases,
300 larger organized structures are formed as shown in Fig. 4, which is similar to the results of
301 HM09.

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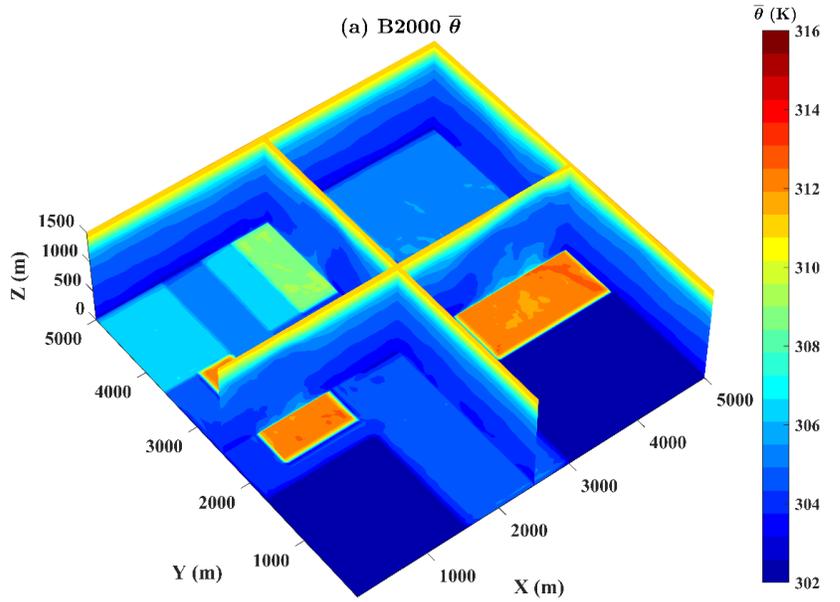
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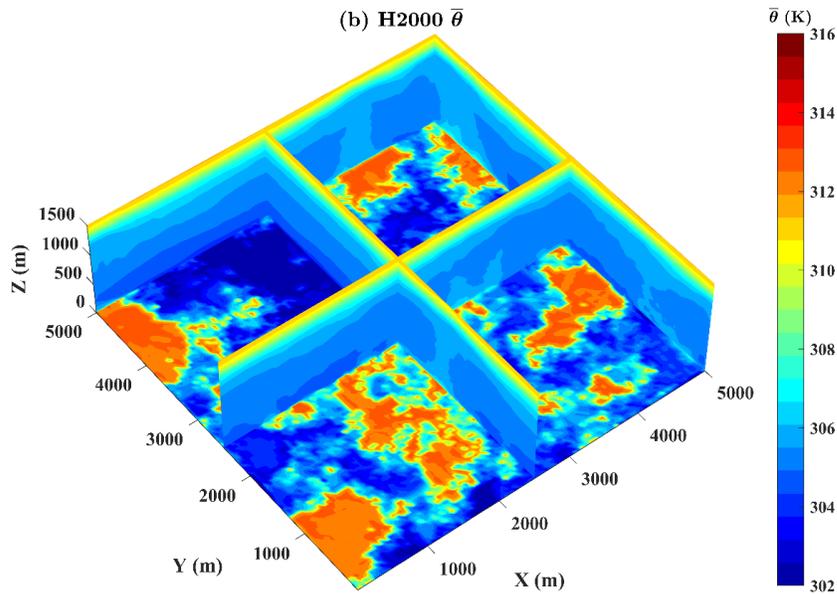
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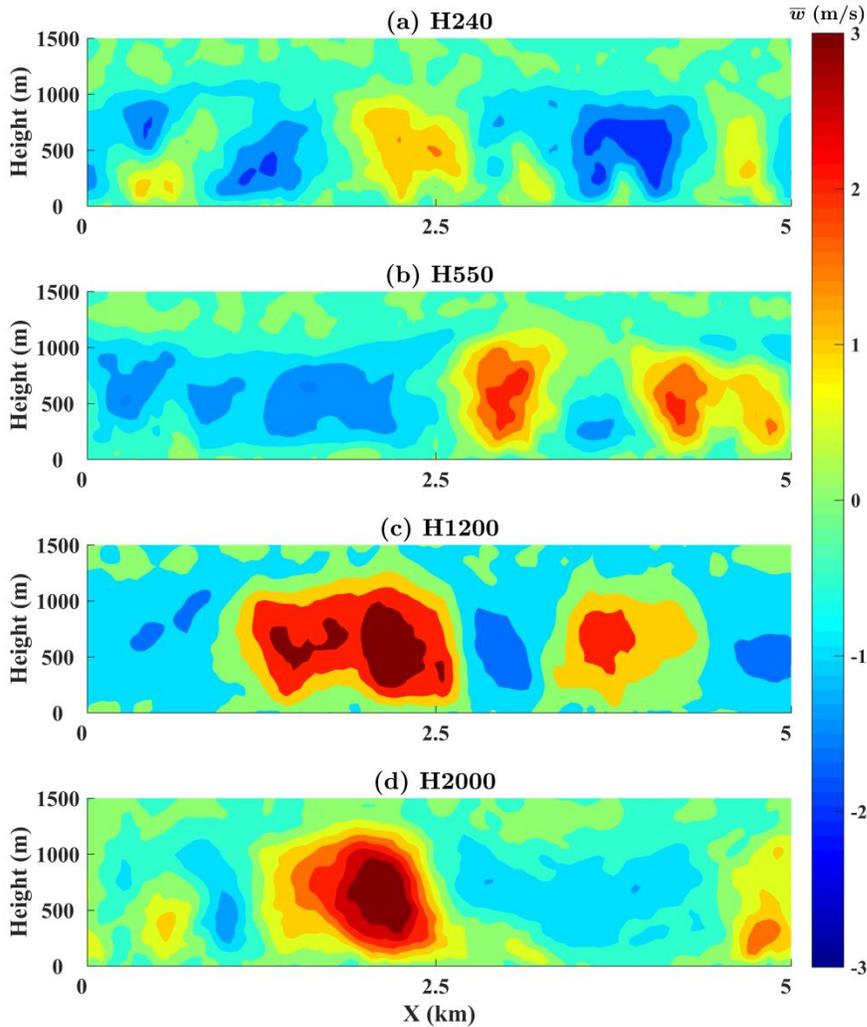


313

314

315

Figure. 3 The temporally averaged potential temperature during the final running hour in case (a) B2000 and (b) H2000. The horizontal slice is at $z = 2$ m.



316

317 **Figure. 4** The cross-sections (x - z) of temporally averaged vertical velocity during the final running hour in H
 318 cases at $y = 3.5$ km.

319

320 Figure 5 shows the vertical profiles of mean potential temperature (θ), σ_u^2 , σ_w^2 , and TKE.
 321 The σ_u^2 and σ_w^2 are normalized by w_*^2 . Clearly, the cases with the smallest heterogeneity scales
 322 (B240 and H240) show structures of a well-mixed CBL. When the heterogeneity scale becomes
 323 larger, θ increases with height in the CBL, especially in cases B2000 and H2000. This is in
 324 agreement with the results in Raasch and Harbusch (2001) and Avissar and Schmidt (1998). One
 325 possible reason is that the asymmetry of circulation within the CBL, i.e., locally restricted
 326 updrafts and more extended weaker downdrafts (Deardorff et al., 1969), leads to an
 327 increase in θ with z within the CBL. Another possible reason is that when the surface
 328 heterogeneity scale is larger, the organized structures generated by the surface heterogeneity
 329 have a larger probability to penetrate deeper into the inversion layer than the smaller eddies and
 330 hence warmer air is brought downward in the CBL by entrainment, which leads to the larger θ

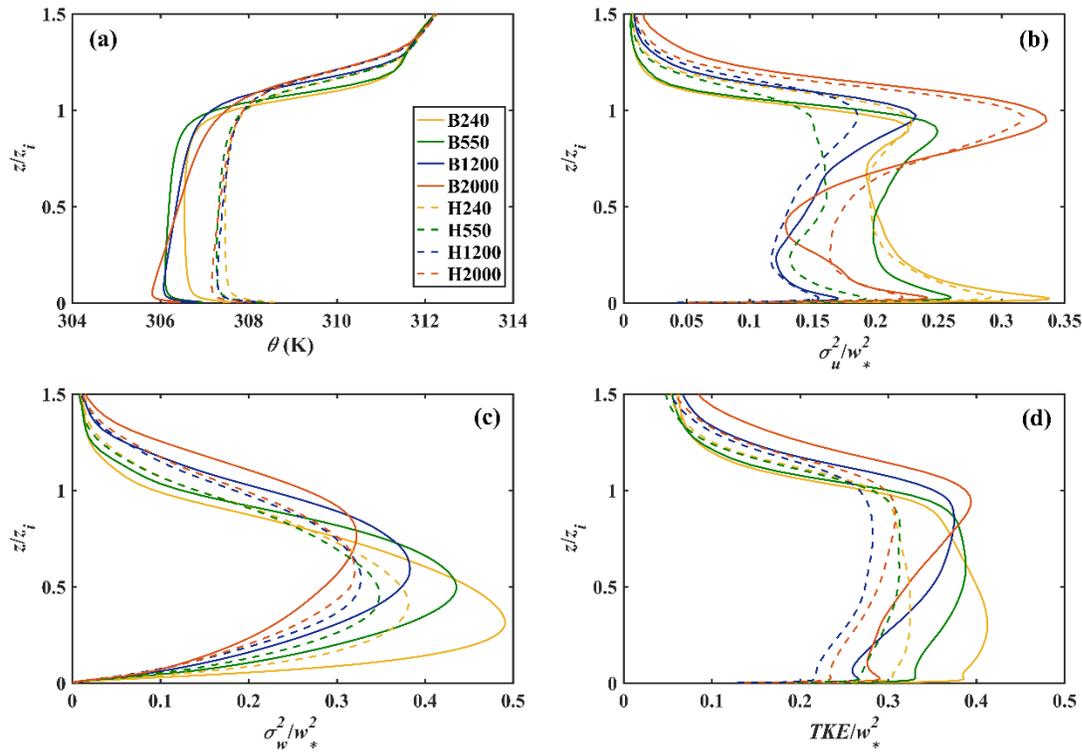
331 with higher height, especially when the surface heterogeneous scale is larger than z_i (Fig. 5a).
 332 The vertical profiles of σ_u^2 and σ_w^2 indicate characteristics similar to many previous studies
 333 (Avisar and Schmidt, 1998; Patton et al., 2005; HM09). Namely, σ_u^2 increases near the surface
 334 and the entrainment zone; whereas, σ_w^2 reaches its maximum value at the middle of mixed layer.
 335 As the heterogeneity scale becomes larger, σ_w^2 in $0 \sim 0.5 z_i$ becomes smaller, which is consistent
 336 with the results in HM09 and Patton et al. (2005). The main explanation is that the larger surface
 337 heterogeneity scale, the more aggregated areas with stronger/weaker heat flux and hence more
 338 clearly separated regions with upward and downward eddies (Patton et al., 2005), which reduces
 339 the σ_w^2 with large surface heterogeneity scale. Similarly, due to this reason in $0 \sim 0.5 z_i$, σ_u^2
 340 generally decreases with increasing heterogeneity scale, which is different from the results in
 341 HM09 and Patton et al. (2005), where σ_u^2 increases with increasing surface heterogeneity scale.
 342 The free convective atmosphere instead of the shear convective atmosphere in HM09 may be
 343 responsible for this difference.

344 In addition, near the entrainment zone, σ_u^2 increases with increasing surface heterogeneity
 345 scale and reaches its maximum value at the heterogeneous scale of 2000 m, which is different
 346 from the trend near the surface. This discrepancy could be related to the observed correlation that
 347 the larger the surface heterogeneity scale, the larger perturbation induced by entrainment and
 348 hence the larger σ_u^2 (Fig. 5b). When the surface heterogeneous scale is larger than z_i , the effects
 349 of entrainment on σ_u^2 are maximum and can reach the surface, which leads to a larger σ_u^2 at the
 350 heterogeneous scale of 2000 m than that of 1200 m (Fig. 5b). Similarly, due to the effects of
 351 entrainment, the height at which the largest σ_w^2 occurs also increases with increasing
 352 heterogeneity scale (Fig. 5c).

353 The profile of TKE is affected by both σ_u^2 and σ_w^2 . In general, its value decreases with
 354 increasing heterogeneity scale (Fig. 5d). However, it should be noted that near the surface (< 0.1
 355 z_i), the TKE is smallest in cases with heterogeneity scale of 1200 m instead of 2000 m due to the
 356 behavior of σ_u^2 .

357 Compared with H cases, θ is smaller in B cases, which is again caused by the different
 358 spatial patterns of roughness length and surface temperature (Fig. 2). For example, in the
 359 simulation of H2000 (Fig. 2e), the hot patches (higher surface temperature) are located in the
 360 center, which induces a relatively smaller extent of weaker downdraughts in the domain.
 361 However, in the simulation of B2000, the hot patches are close to the borders (Fig. 2a), which
 362 leads to more extended weaker downdraughts in the domain. Because of the more extended
 363 weaker downdraughts, the potential temperature is smaller in the simulation of B2000.

364



365

366 **Figure. 5** The vertical profiles of ABL statistics: (a) potential temperature (θ), (b) variance of horizontal
 367 velocity (σ_u^2), (c) variance of vertical velocity (σ_w^2), and (d) TKE. All data are averaged in the final hour of
 368 simulation and (b, c, d) are normalized by the convective velocity. The boundary-layer height (z_i) is defined as
 369 the height where the minimum (negative) value of heat flux is found, and the vertical scale is normalized by z_i
 370 in each case.

371

4.1.2 The flux imbalance

372

Figure 6(a) shows the vertical profiles of flux imbalance magnitude in different cases.
 373 Clearly, the flux imbalance magnitude increases with increasing height in both cases, which is
 374 consistent with previous studies (Kanda et al., 2004; Steinfeld et al., 2007; Huang et al., 2008;
 375 Zhou et al., 2018). It is also evident that the curves tend to cluster together at lower heights (z/z_i
 376 < 0.02). This is due to the fact that the parameterized subgrid-scale turbulence becomes
 377 important and the subgrid-scale flux dominates.

378

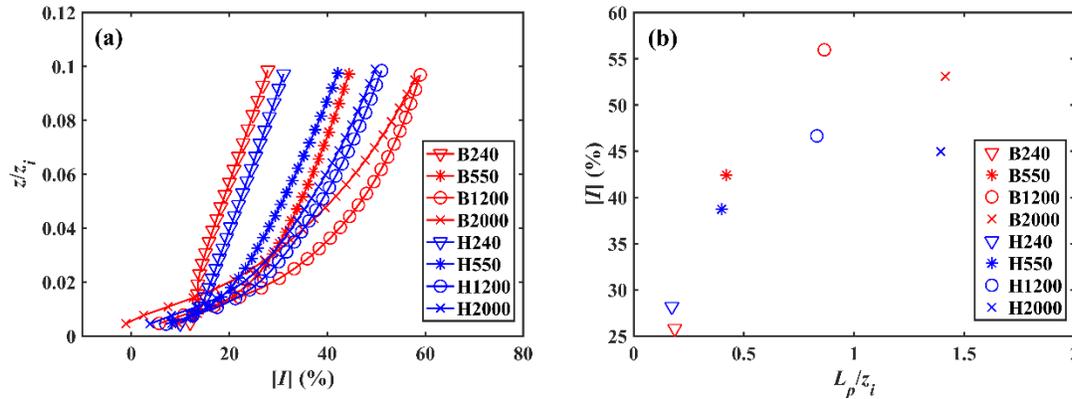
To examine the relation between the flux imbalance magnitude and the heterogeneity
 379 scale, the results at 112 m ($z/z_i \approx 0.08$) are shown as an example where the effects of subgrid
 380 parameterization should be very small. We choose to analyze the flux imbalance at a fixed z
 381 instead of a fixed z/z_i here because in practice all EC measurements have a fixed z . Overall the
 382 flux imbalance magnitude increases with increasing heterogeneity scales, as shown in Fig. 6b.
 383 This is consistent with field observations. For example, Stoy et al. (2013) and Xu et al. (2017)
 384 found that the flux imbalance magnitude increases with increasing landscape-level and footprint
 385 variability, respectively. This can be understood because the larger the heterogeneity scale is, the
 386 larger the circulations induced by surface heterogeneity become. These large-scale motions are

387 less likely to be adequately sampled in a finite average period and hence a larger flux imbalance
388 occurs.

389 Interestingly, when the heterogeneity scale becomes larger than z_i , the flux imbalance
390 magnitude seems to decrease, especially in case B2000 (Fig. 6b). That is, the flux imbalance
391 magnitude reaches its maximum value when the surface heterogeneity scale is about z_i . Close
392 inspection of Fig. 6a reveals that this is the case nearly everywhere as long as $z/z_i > 0.02$. The
393 further reduction of flux imbalance magnitude with increasing heterogeneity scale is probably
394 because the large organized structures generated by the heterogeneity scale of 2000 m can
395 penetrate deeper into the inversion layer and hence warmer air is brought downward into the
396 CBL by entrainment. This additional heat flux caused by entrainment can reach the surface (see
397 Fig. 5a, b), which reduces the flux imbalance magnitude.

398 In comparison, the flux imbalance in B cases are generally larger than those in H cases.
399 As discussed in Sect. 4.1.1, the smaller TKE and vertical turbulent heat flux induced by the more
400 extended weaker downdraughts in B cases are responsible for this difference.

401



402

403 **Figure. 6** (a) The vertical profiles of flux imbalance magnitude in different cases; and (b) the flux imbalance
404 magnitude at 112 m as a function of surface heterogeneity scales (L_p) normalized by the boundary layer height
405 (z_i).

406 4.2 Interpretation of flux imbalance using the cospectral model

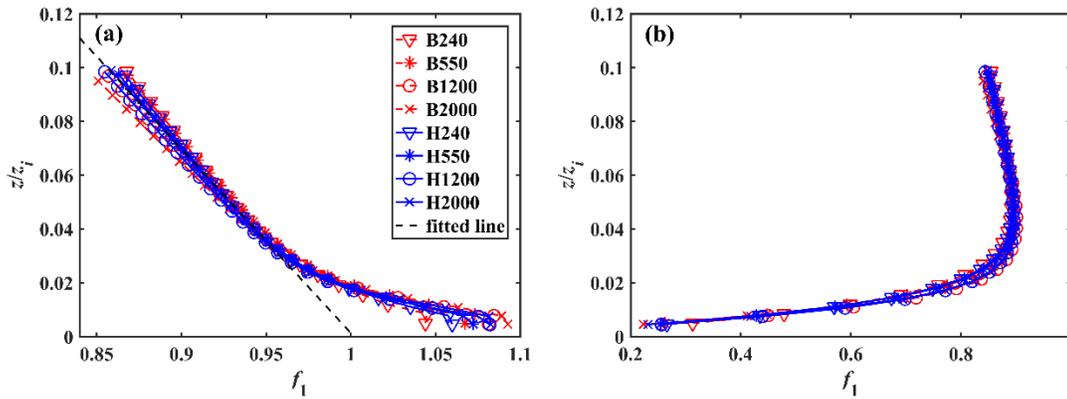
407 The conceptual model described in Sect. 2.3 connects the flux imbalance with f_1 and f_2 ,
408 where f_1 represents the ratio of “flux at a certain height” to the surface flux and f_2 represents the
409 role of large eddies in inducing flux imbalance. Specifically, the flux imbalance magnitude is
410 expected to increase with decreasing f_1 and f_2 . In the following, we investigate the relations
411 between flux imbalance magnitude and f_1 and f_2 , as well as their controlling factors.

412 4.2.1 f_1

413 The f_1 represents the ratio of the “flux at a certain height” to the surface flux. In the
414 following, the “flux at a certain height” is calculated using the spatial method, i.e., it is the spatial
415 turbulent heat flux calculated at a certain height. Figure 7(a) shows the calculated f_1 at different
416 heights when the subgrid-scale heat flux is included. As can be seen, f_1 decreases with increasing
417 heights in all cases when the subgrid-scale heat flux is included (Fig. 7a). Based on Eq. (11), the
418 decrease of f_1 with increasing heights leads to the increase of flux imbalance magnitude, which is

419 consistent with Fig. 6(a). However, there is virtually no difference between the two cases and
 420 also between simulations with different heterogeneity scales. As a result, Fig. 7(a) cannot explain
 421 the variations of flux imbalance with respect to heterogeneity scales (Fig. 6b).

422 Note again that f_1 has large uncertainty at lower heights due to the role of subgrid
 423 turbulence parameterization (c.f. Fig. 7a where the subgrid-scale heat flux is included and Fig. 7b
 424 where the subgrid-scale heat flux is excluded). For example, near the surface the f_1 values
 425 become even larger than 100% when the subgrid-scale flux is included (Fig. 7a), suggesting that
 426 the f_1 values at lower heights are less trustable. To avoid the effect of subgrid turbulence
 427 parameterization, we will focus on the results in the range of $0.03 < z/z_i < 0.1$ in the following.



428
 429 **Figure. 7** The vertical profiles of f_1 with (a) the subgrid-scale heat flux included and (b) the subgrid-scale heat
 430 flux excluded. The black dash line is the fitted line (see Eq. 14).

431 4.2.2 f_2

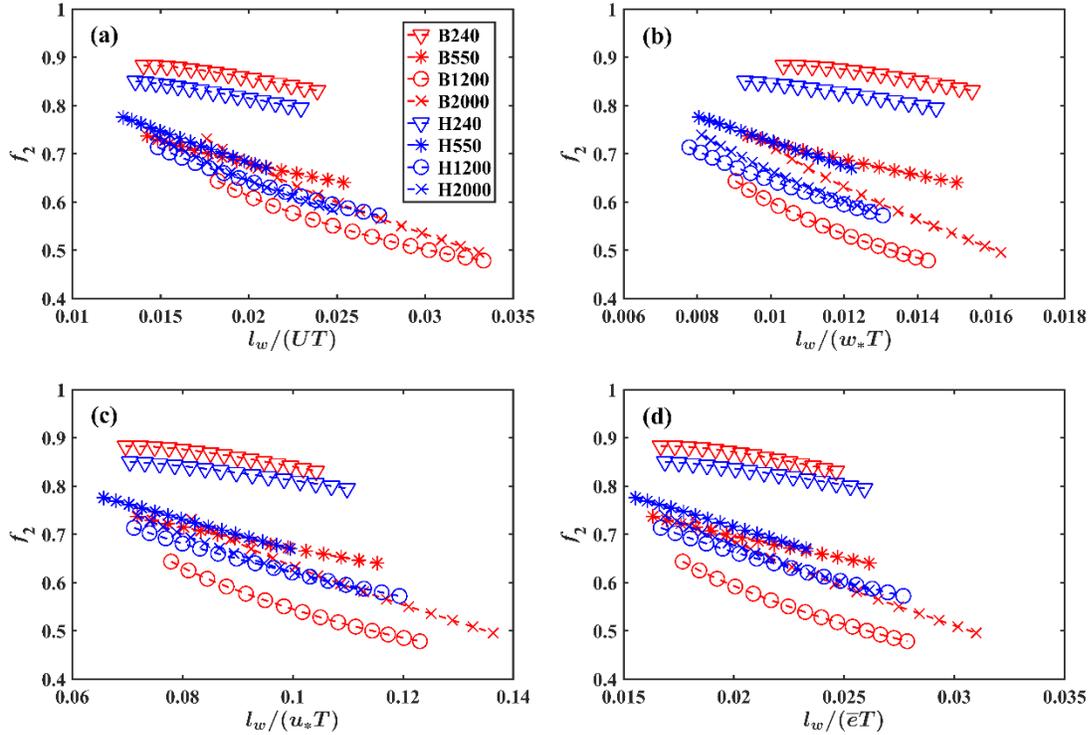
432 In this section, we examine the relation between f_2 , which is inferred from $(1-[I])/f_1$, and
 433 different turbulent length scales that might characterize k_{ec}/k_p . Note again that k_{ec} is the critical
 434 wavenumber of the EC method and we estimate k_{ec} as $1/(uT)$, where u is a velocity scale that can
 435 be represented U , w_* , u_* and \bar{e} , and T is the averaging period. The k_p is the peak wavenumber
 436 that might be estimated from the integral length scale of vertical velocity (l_w) or the integral
 437 length scale of potential temperature (l_θ). In the following we compare all these velocity and
 438 length scales.

439 Figure 8 shows the results in the range of $0.03 < z/z_i < 0.1$ when k_{ec}/k_p is represented by
 440 $l_w/(uT)$ with different velocity scales. One can see that f_2 generally decreases with increasing
 441 $l_w/(uT)$, consistent with Eq. (11), but with large scatter. Similar results are also found using
 442 $l_\theta/(uT)$ and hence those results are not shown. The large scatter suggests that additional variables
 443 (e.g., z_i/L) should be considered to include the effects of atmospheric stability on turbulent
 444 coherent structures, which are not accounted for by Eq. (11). Similar to previous studies over
 445 homogeneous surfaces (Schalkwijk et al., 2016; Huang et al., 2008), we also found a reasonably
 446 strong correlation between the flux imbalance magnitude and $-z_i/L$ but over heterogeneous
 447 surfaces: the larger the $-z_i/L$, the smaller f_2 and thus the larger the flux imbalance magnitude
 448 (Fig. 9).

449 Therefore, we further examine the correlations between the f_2 and $-z_i/L \times l_w/(uT)$ (Fig.
 450 10). The choice of $-z_i/L \times l_w/(uT)$ is motivated by the fact that the atmospheric stability is likely
 451 to influence the largest eddies and may be treated as an adjustable factor to k_p through altering l_w .

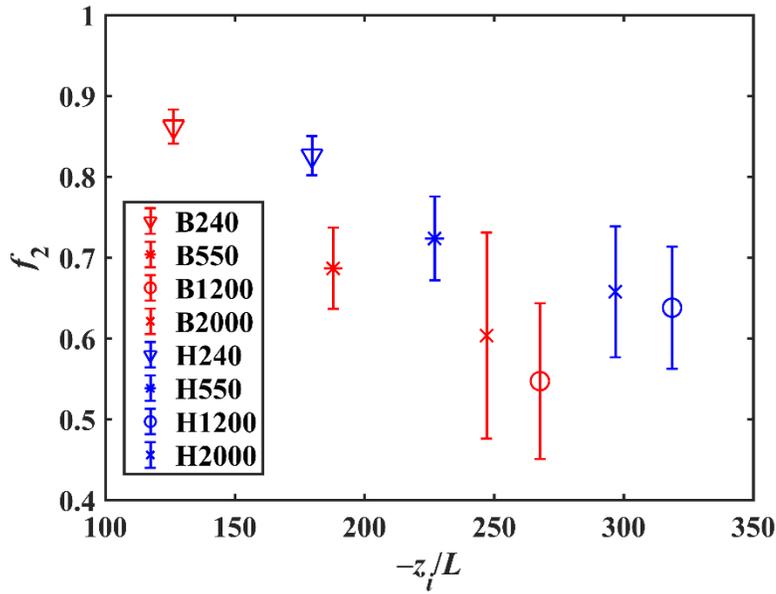
452 Compared to Fig. 8, the negative correlations are much higher in Fig. 10 than those in Fig. 8 (see
 453 the R^2 values in Table 3). The better correlations confirm that $-z_i/L$ plays an important role in
 454 modulating the flux imbalance over heterogeneous surfaces.

455 Among all the velocity scales, using $-z_i/L \times l_w/(UT)$ yields the highest R^2 value (Table 3),
 456 indicating that the mean horizontal velocity U better characterizes k_{ec} . On the other hand, using l_θ
 457 leads to quite parcular relations and very larger scatter (and thus much smaller R^2 values, not
 458 shown here) compared to using l_w (Fig. 11), implying that l_w is a better choice to represent k_p at
 459 least in our cases.



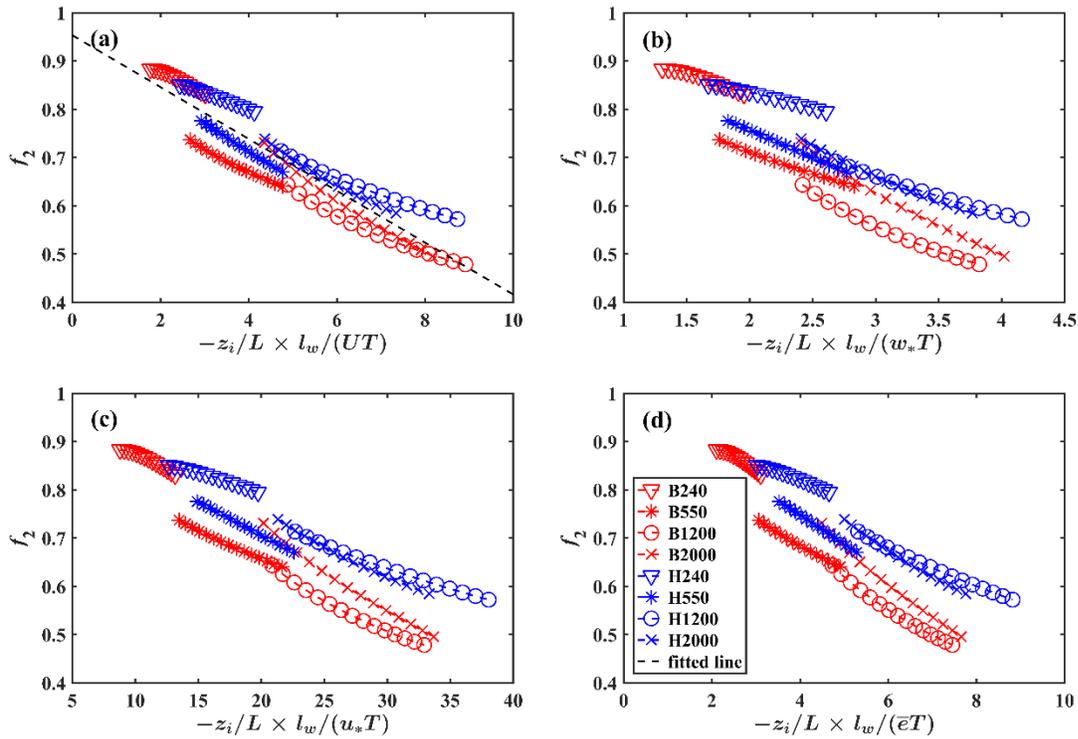
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461 **Figure. 8** The relations between f_2 and (a) $l_w/(UT)$, (b) $l_w/(w_*T)$, (c) $l_w/(u_*T)$, and (d) $l_w/(\bar{\epsilon}T)$. Only results in the
 462 range of $0.03 < z/z_i < 0.1$ are shown.



463

464 **Figure. 9** The f_2 as a function of $-z_i/L$. The marker indicates the mean of results in the range of $0.03 < z/z_i < 0.1$
 465 and the bar indicates the range.



466

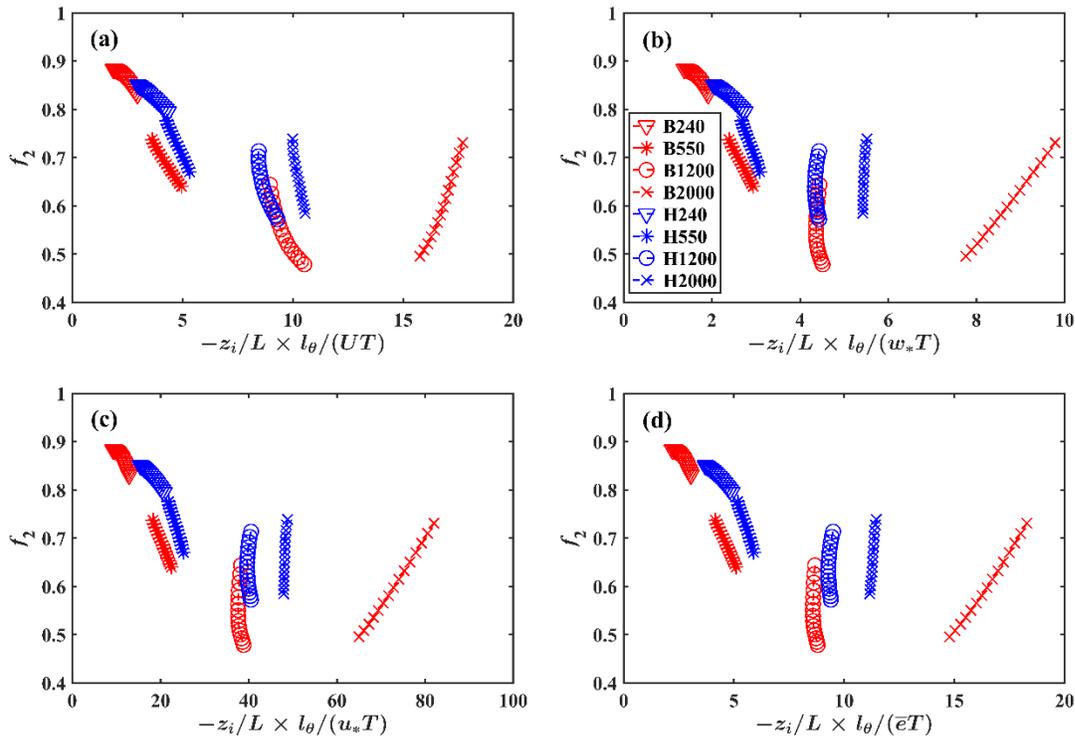
467 **Figure. 10** The relations between f_2 and (a) $-z_i/L \times l_w/(UT)$, (b) $-z_i/L \times l_w/(w*T)$, (c) $-z_i/L \times l_w/(u*T)$, and (d) $-z_i/L \times l_w/(\bar{e}T)$. Only results in the range of $0.03 < z/z_i < 0.1$ are shown.

469

470 **Table 3.** The slopes (K), intercepts (C) and coefficients of determination (R^2) between the f_2 and $l_w/(uT)$ (Fig.
471 8) and $-z_i/L \times l_w/(uT)$ (Fig. 10).

case		$l_w/(UT)$	$l_w/(w_*T)$	$l_w/(u_*T)$	$l_w/(eT)$	$-z_i/L \times l_w/(UT)$	$-z_i/L \times l_w/(w_*T)$	$-z_i/L \times l_w/(u_*T)$	$-z_i/L \times l_w/(\bar{e}T)$
B	K	-19.26	-12.11	-5.47	-22.27	-0.06	-0.16	-0.02	-0.07
	C	1.11	0.83	1.21	1.17	0.96	1.09	1.02	1.03
	R^2	0.78	0.17	0.69	0.62	0.95	0.95	0.97	0.96
H	K	-15.23	-3.89	-3.26	-13.25	-0.05	-0.12	-0.01	-0.05
	C	1.00	0.75	1.00	0.99	0.94	1.03	0.98	0.98
	R^2	0.65	0.08	0.53	0.50	0.93	0.92	0.94	0.94
B + H	K	-16.86	-10.64	-4.54	-18.66	-0.05	-0.14	-0.01	-0.06
	C	1.04	0.82	1.12	1.10	0.95	1.06	0.99	0.98
	R^2	0.74	0.19	0.64	0.58	0.92	0.90	0.89	0.87

472



473

474 **Figure. 11** The relations between f_2 and (a) $-z_i/L \times l_\theta/(UT)$, (b) $-z_i/L \times l_\theta/(w_*T)$, (c) $-z_i/L \times l_\theta/(u_*T)$, and (d) $-z_i/L \times l_\theta/(\bar{e}T)$. Only results in the range of $0.03 < z/z_i < 0.1$ are shown.
475

476

4.2.3 A diagnostic equation for $[I]$

477

Given that the behavior of f_1 with z is quite predictable from Fig. 7, f_1 may be parameterized as follows:
478

$$f_1 = a \frac{z}{z_i} + b, \quad (14)$$

479 where $a = -1.46$ and $b = 1.0$ are fitted parameters (see the fitted line in Fig. 7). Again, to avoid the
 480 effect of subgrid turbulence parameterization, only data between in the range of $0.03 < z/z_i < 0.1$
 481 are used to fit the parameters in Eq. (14). The linear decrease of turbulent heat flux in the CBL is
 482 a well-established result (Garratt 1992), and our results confirms this. Note that we did not *a*
 483 *priori* specify the value of b . The fact that $b = 1.0$ is obtained from the fitting demonstrates that
 484 the linear decreasing trend in the range of $0.03 < z/z_i < 0.1$, when extrapolated to the surface,
 485 recovers the surface heat flux.

486 Similarly, given the nearly linear relationship between f_2 and $-z_i/L \times l_w/(UT)$ (here we use
 487 U due to the largest R^2 value it yields as shown in Table 3 and also the fact that its measurement
 488 is often available), we can parameterize f_2 as

$$f_2 = K \frac{z_i}{L} \frac{l_w}{UT} + C, \quad (15)$$

489 where $K = -0.05$ and $C = 0.95$ are fitted parameters (see the fitted line in Fig. 10a). Based on
 490 Eqs. (7), (14), (15), the flux imbalance magnitude can be parameterized as follows:

$$[I] = 1 - \left[a \frac{z}{z_i} + b \right] \left[K \frac{z_i}{L} \frac{l_w}{UT} + C \right]. \quad (16)$$

491 In practice, some of the inputs for Eq. (16) are not always available even if the
 492 coefficients a , b , K , and C are provided by fitting to LES results. For example, the calculation of
 493 z_i/L requires the boundary layer height and the surface heat flux. The former is rarely measured
 494 in the field while the latter is what the EC aims to measure and thus is unknown *a priori*. From
 495 this point view, Eq. (16) is a *diagnostic* equation and cannot be used as a *prognostic* equation to
 496 compute the flux imbalance. We note that this is also the case for other parameterizations in the
 497 literature (see e.g., Huang et al., 2008).

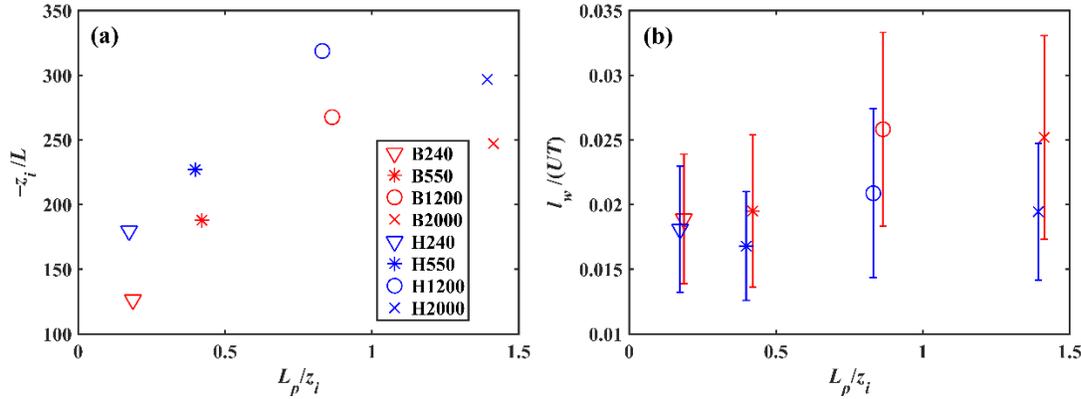
498 5 Discussion

499 5.1 The relation between the surface heterogeneity scale and flux imbalance

500 Our results demonstrate that the following variables primarily affect the flux imbalance:
 501 the integral length scale of vertical velocity l_w , the atmospheric stability parameter z_i/L , the mean
 502 horizontal velocity U , and the average period T , when the cospectral model is employed to
 503 interpret the flux imbalance. Among these four variables, l_w determines the maximum size of
 504 large-scale turbulent structures (i.e., the large eddies), z_i/L affects the shape of these turbulent
 505 structures and also the cospectrum form, i.e., the flux distribution among different eddy sizes,
 506 whereas U and T determine the number of large eddies that can be sampled by the EC sensors in
 507 a finite averaging period.

508 These characteristic variables are strongly affected by the surface heterogeneity. For
 509 example, one can infer from Fig. 9 that z_i/L is dependent on the surface heterogeneity scale.
 510 Figure 12a shows this more clearly. In addition, Fig. 12b shows the dependence of $l_w/(UT)$ on the
 511 surface heterogeneity scale. Due to the dependence of these characteristic variables on the
 512 surface heterogeneity scale, the flux imbalance is shown to be affected by surface heterogeneity
 513 in both our simulations (Fig. 6b) and previous observational studies (Stoy et al., 2013; Xu et al.,
 514 2017). Our results demonstrate that although the pdf of surface characteristics remains identical,
 515 landscapes with different characteristic length scales can have vastly different flux imbalance

516 magnitude. This may explain why the relation between the surface heterogeneity and the flux
 517 imbalance observed in the field data shows large variability (Stoy et al., 2013; Xu et al., 2017).



518
 519 **Figure 12** The (a) $-z_i/L$ and (b) $I_w/(UT)$ as a function of L_p/z_i . The marker in (b) indicates the mean of results
 520 in the range of $0.03 < z/z_i < 0.1$ and the bar in (b) indicates the range.

521 5.2 The relations between the Eq. (16) and related factors

522 As stated in the introduction, previous studies have reported relations between flux
 523 imbalance and various factors (see Table 1). In this section, we use the conceptual model (i.e.,
 524 Eq. 16) to explain these reported relations in the literature imbalance. Based on Eq. (16), it can
 525 be easily seen that the flux imbalance I increases with increasing z and $-z_i/L$, decreasing U (when
 526 u is represented by U) and TKE (when u is represented by TKE), and shorter T . Due to the
 527 increasing of I with z and $-z_i/L$, I also increases with the increasing of $-z/L$. In addition, as
 528 discussed in Sect. 5.1 and shown in Sect. 4.1.2 (Fig. 6b), overall the flux imbalance I increases
 529 with increasing surface heterogeneity scale. Because the $-z_i/L$ can be expressed as follows:

$$530 \quad -\frac{z_i}{L} = k \left(\frac{w_*}{u_*} \right)^3, \quad (17)$$

531 where k is the von-Karman constant. Based on Eq. (17) and Eq. (16), we can also see that the
 532 flux imbalance I increases with decreasing u_*/w_* . These results inferred from our conceptual
 533 model are consistent with previous studies (see Table 1).

534 Furthermore, we can express Eq. (9) as follows:

$$535 \quad f_2 = 1 - \frac{\int_0^{k_{ec}} F_{w\theta}(k, z) dk}{\int_0^{\infty} F_{w\theta}(k, z) dk} = 1 - \frac{1}{R_{w\theta}(z)\sigma(w)\sigma(\theta)} \int_0^{k_{ec}} F_{w\theta}(k, z) dk, \quad (18)$$

536 where $R_{w\theta}(z)$ is the correlation coefficient between w and θ at z ; $\sigma(w)$ and $\sigma(\theta)$ are standard
 537 deviations of w and θ , respectively. A larger phase difference between w and θ results in a
 538 smaller $R_{w\theta}$ and thus a smaller f_2 (Eq. 18), which leads to a larger I based on Eq. (7). This
 539 conclusion is consistent with results in McGloin et al. (2018) and Gao et al. (2017).

540 In summary, with the proposed diagnostic equation for I (i.e., Eq. 16), the relations
 541 between the flux imbalance and various factors reported in the literature (see Table 1) can be all
 542 explained.

541 5.3 A comparison of different parameterizations for flux imbalance

542 Several energy balance closure parametrization schemes have been proposed (Panin et
 543 al., 1998; Huang et al., 2008; Panin and Bernhofer, 2008). In this paper, we also propose a
 544 conceptual model to explain the flux imbalance. Here, we compare these models and explain
 545 their advantages and limitations.

546 Based on LES runs over homogeneous surfaces, Huang et al. (2008) parametrized the
 547 flux imbalance as a function of non-dimensionalized turbulent velocity scales as

$$[I] = \left[\exp^{(a+b\frac{u_*}{w_*})} + c \right] \left[1.1 + d \left(\frac{z}{z_i} + f \right)^2 \right]^{0.5}, \quad (19)$$

548 where $a = 4.2$, $b = -16$, $c = 2.1$, $d = -8.0$ and $f = -0.38$ are fitted parameters. It has been
 549 found the biggest shortcoming of this empirical parametrization scheme is that it cannot be
 550 applied directly to heterogeneous surfaces (Zhou et al., 2018). Considering that surface
 551 heterogeneity is one of the main contributors to flux imbalance, the applicability of this
 552 parametrization scheme is greatly limited. In addition, this parameterization is developed based
 553 on LES data above the surface layer ($0.3 < z/z_i < 0.5$) and hence its applicability for analyzing
 554 real observations, which are mostly collected in the surface layer, is questionable. This may be
 555 why field observations have reported failure of using this relation to capture flux imbalance
 556 (Eder et al., 2014). Lastly, as alluded to earlier, this parameterization cannot be used as a
 557 prognostic equation since one of the required inputs is the surface heat flux for computing w_* .

558 To consider the effect of surface heterogeneity on the flux imbalance, Panin and
 559 Bernhofer (2008) directly related the effective surface roughness length (z_0^{eff}) and surface
 560 heterogeneity scale (i.e., L^{eff}) to the flux imbalance and proposed the following parameterization

$$EBR = 1 - [I] = K \frac{z_0^{eff}}{L^{eff}} + C, \quad (20)$$

561 where EBR is the energy-balance closure ratio (the ratio of the sum of turbulent heat
 562 fluxes to the available energy), and K and C are empirical constants. Compared to the
 563 parameterization by Huang et al. (2008), the biggest advantage of this empirical parametrization
 564 scheme is that it explicitly considers the effects of surface heterogeneity and hence it can be used
 565 over heterogeneous surfaces. However, there are also some limitations associated with this
 566 parameterization. First, only the effect of momentum roughness length is included. Second, as
 567 shown in Fig. 6, even the same surface heterogeneity scale can yield very different flux
 568 imbalance magnitude. The large variation of flux imbalance magnitude suggests that the
 569 empirical constants (i.e., K and C) may be varying across sites and hence it is not surprising that
 570 the performance of this model also varies across sites (Eder et al., 2014). Last, the atmospheric
 571 stability and its interaction with surface heterogeneity can strongly affect turbulent organized
 572 structures, which further alters the flux imbalance. Hence, only considering the surface
 573 heterogeneity scale cannot correctly capture the flux imbalance under a variety of atmospheric
 574 stability conditions.

575 Compared to the previous two models, our model is derived from the cospectrum and
 576 hence is more physically based. It should be pointed out that the surface heterogeneity scale is
 577 not explicit in our model. This is because the model is constructed based on the assumption that
 578 large-scale organized turbulent structures lead to the flux imbalance. The effect of surface

579 heterogeneity on flux imbalance is implicitly included in our model by considering the impact of
 580 surface heterogeneity on characteristic variables of turbulent flows, especially those related to
 581 large eddies.

582 There are also some limitations of our model. Most importantly, our model is based on
 583 the assumption that the flux imbalance is only caused by the large eddies. Therefore, the flux
 584 imbalance caused by other factors cannot be explained by our model. In addition, our model is
 585 also a diagnostic model and cannot be directly applied in practice to calculate the flux imbalance.
 586 Lastly, our model is constructed with data in the range $0.03 < z/z_i < 0.1$. While this range is much
 587 lower than that in Huang et al. (2008), the lowest limit ($z/z_i = 0.03$) is still higher than the typical
 588 measurement height in the field. Whether our model can capture the behavior of flux imbalance
 589 near the surface ($z/z_i < 0.03$) needs to be investigated using higher-resolution LES simulations in
 590 the future.

591 **6 Conclusions**

592 This study analyzes the relationship between the surface heterogeneity scale and the flux
 593 imbalance over heterogeneous landscapes in a dry convective boundary layer. The main
 594 conclusions are summarized as follows:

595 (1) The surface heterogeneity scale strongly affects the flux imbalance even when the
 596 pdfs of surface characteristics are the same. The flux imbalance magnitude initially increases
 597 with increasing surface heterogeneity scale and reaches its peak value when the surface
 598 heterogeneity scale is similar to the boundary-layer height z_i . When the surface heterogeneity
 599 scale is larger than z_i , the organized structures can penetrate deeper into the inversion layer and
 600 hence warmer air is brought downward in the CBL by entrainment. This additional heat flux
 601 reduces the flux imbalance magnitude to a certain degree.

602 (2) A conceptual model for the flux imbalance is proposed to better understand the flux
 603 imbalance induced by the large eddies. Based on this conceptual model, we find that z_i/L , l_w , U ,
 604 and T are the key variables that control the flux imbalance. Among these three variables, the l_w
 605 determines the maximum size of large eddies; the z_i/L represents the form of these large eddies
 606 or the cospectrum form, i.e., the flux distribution among different eddy sizes; the U and T
 607 determine the numbers of these large eddies can be sampled by the EC in a finite averaging
 608 period.

609 (3) Assuming that the flux imbalance is mainly caused by the inadequate sampling of
 610 large eddies, a diagnostic equation for the flux imbalance magnitude is proposed as $[I] = 1 - [az/z_i$
 611 $+ b][-K \times z_i/L \times l_w/UT + C]$, where a , b , K , and C are empirical constants and b should in theory
 612 equal to unity. Compared to the other empirical and semi-empirical models, our model is derived
 613 from the cospectrum and hence is more physically based. Moreover, the relations between the
 614 flux imbalance and various factors reported in the literature can be explained by our model. Note
 615 that our model is diagnostic because it requires *a priori* knowledge of the surface heat flux. In
 616 addition, it also requires boundary-layer height as an input, which is not always available. It is
 617 also pointed out that this model is constructed with data in the range $0.03 < z/z_i < 0.1$.

618 Our study has a few limitations that should be pointed out. First, it is important to point
 619 out that our study does not consider latent heat flux, whose behavior might be different from that
 620 of sensible heat flux, especially in the low frequency (Cava et al., 2008; Detto et al., 2008;
 621 Huang et al., 2009; Li and Bou-Zeid, 2011; Li et al., 2012; Cancelli et al., 2014; Charuchittipan

622 et al., 2014; Gao et al., 2017). Second, we only examine the flux imbalance under free
 623 convective conditions and the effect of wind remains to be investigated. Last, it is noted that we
 624 use the control volume method (i.e., a regular control volume) to calculate the flux imbalance.
 625 However, in the field, the footprint of EC is not a rectangle and varies with the wind and
 626 atmospheric stability. The effect of this mismatch between the regular control volume and the
 627 irregular footprint (Metzger, 2018) on flux imbalance needs to be investigated in the future.

628 **Appendix A: The derivation of Eq. (11)**

629 In this appendix, the flux imbalance is deduced by using the cospectral model of Lee et
 630 al. (2005), which can be expressed as follows:

$$F_{w\theta}(k) = \frac{A_{w\theta}}{k_p \left[1 + m_{w\theta} \left(\frac{k}{k_p} \right)^{2\mu} \right]^{\frac{1}{2\mu} \frac{m_{w\theta}+1}{m_{w\theta}}}}, \quad (21)$$

631 where $A_{w\theta}$ is a normalization parameter, k_p is the peak wavenumber, and $m_{w\theta}$ and μ are the
 632 (inertial subrange) slope parameter and broadness parameter, respectively. Substituting Eq. 21
 633 into the Eq. (9), the flux imbalance can be expressed as:

$$f_2 = 1 - \frac{\frac{D\Gamma\left(\frac{1}{2\mu}, 0\right) - D\Gamma\left(\frac{1}{2\mu}, \frac{m+1}{2\mu} \left(\frac{k_{ec}}{k_p}\right)^{2\mu}\right)}{(1+m)^{\frac{1}{2\mu}}}}{\frac{D\Gamma\left(\frac{1}{2\mu}, 0\right)}{(1+m)^{\frac{1}{2\mu}}}} = \frac{\Gamma\left(\frac{1}{2\mu}, \frac{m+1}{2\mu} \left(\frac{k_{ec}}{k_p}\right)^{2\mu}\right)}{\Gamma\left(\frac{1}{2\mu}, 0\right)}, \quad (22)$$

634 where

$$D = 2^{\frac{1}{2\mu}-1} a \mu^{\frac{1}{2\mu}-1}, \quad (23)$$

$$\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt. \quad (24)$$

635 Eq. (22) is identical to the Eq. (11) in the main text. Due to the decrease of $\Gamma(s, x)$ with
 636 increasing x , f_2 is expected to decrease as k_{ec}/k_p increases.

637 **Appendix B: Different cospectral models and the derived f_2**

638 In this appendix, different cospectral models are used to derive the flux imbalance.

639 1 The cospectral model of Katul et al. (2013)

640 Katul et al. (2013) provided a simple cospectral model, which can be expressed as
 641 follows:

$$F_{w\theta}(k) = \begin{cases} \frac{\varepsilon^{-\frac{1}{3}} k_p^{-\frac{7}{3}}}{C_1} \{dT C_{ww} + (1 - C_2) \beta C_{\theta\theta}\}, & k > k_p \\ \frac{\varepsilon^{-\frac{1}{3}} k^{-\frac{7}{3}}}{C_1} \{dT C_{ww} + (1 - C_2) \beta C_{\theta\theta}\}, & k \leq k_p \end{cases}, \quad (25)$$

642 where ε is the TKE dissipation rate; k_p is the peak wavenumber; dT and β are the air
 643 temperature gradient and buoyancy parameter, respectively; and C_2 , C_{ww} and $C_{\theta\theta}$ are constants.
 644 Substituting Eq. 25 into the Eq. (9), the flux imbalance leads to:

$$f_2 = 1 - \frac{\frac{k_{ec} \varepsilon^{-\frac{1}{3}} k_p^{-\frac{7}{3}}}{C_1} \{\Gamma C_{ww} + (1 - C_2) \beta C_{\theta\theta}\}}{\frac{7}{4} k_p^{-\frac{4}{3}} \frac{\varepsilon^{-\frac{1}{3}}}{C_1} \{\Gamma C_{ww} + (1 - C_2) \beta C_{\theta\theta}\}} = 1 - \frac{4}{7} \frac{k_{ec}}{k_p}. \quad (26)$$

645 Clearly, f_2 decreases with increasing k_{ec}/k_p .

646 2 The cospectral model of Massom and Lee (2002)

647 Massom and Lee (2002) gave another simple expression for cospectrum:

$$F_{w\theta}(k) = \frac{2}{\pi} \frac{1}{k_p [1 + (k/k_p)]^2}. \quad (27)$$

648 Substituting Eq. 27 into Eq. (9), f_2 can be expressed as follows:

$$f_2 = \frac{1}{\left(1 + \frac{k_{ec}}{k_p}\right)}. \quad (28)$$

649 It is clear that in this model f_2 also decreases with increasing k_{ec}/k_p .

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